Cosmic Microwave Background, Accelerating Universe and Inhomogeneous Cosmology

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Abstract

We consider a cosmology in which a spherically symmetric large scale enhancement or a void are described by an inhomogeneous metric and gravitational equations, including matter pressure and a cosmological constant. For a flat matter dominated universe the inhomogeneous equations lead to luminosity distance and Hubble constant formulas that depend on the location of the observer and a deceleration parameter that can differ significantly from the FLRW result. The deceleration parameter q_0 can be interpreted as being $q_0 < 0$ in a FLRW universe and be $q_0 > 0$ ($q_0 = 1/2$ for a flat matter dominated universe) as inferred from the inhomogeneous perturbation enhancement that is embedded in a FLRW universe. The CMB temperature fluctuations across the sky can be unevenly distributed in the northern and southern hemispheres in the inhomogeneous matter dominated solution, in agreement with the analysis of the WMAP power spectrum data by several authors. The model can possibly explain the anomalous alignment of the quadrupole and octopole moments observed in the WMAP data.

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1 Introduction

The problem of how to explain the accelerating expanding universe given the SNe Ia supernova data [1, 2] in combination with the WMAP data has led to a host of solutions, ranging from modified gravity theories to a quintessence field and a cosmological constant [3, 4]. In the following we shall take the position that the homogeneous and isotropic Friedmann-Lemaître-Robertson-Walker (FLRW) cosmology is the most symmetric model which describes many of the basic features of the data.

However, important physical features may not be explained by this first approximation and a more complete description is given by an inhomogeneous cosmology [5]. We shall investigate a universe in which a spherically symmetric perturbation enhancement or void is embedded in an asymptotic FLRW universe. We find that the inhomogeneities permitted by observation can lead to a reinterpretation of the luminosity distance d_L of a cosmological source in terms of its red shift z. The time evolution and the expansion rate of the inhomogeneous universe can lead to intrinsic effects such as cosmic variance at large angles and long-wavelength perturbations not described by a FLRW homogeneous and isotropic universe. Therefore, the inference from the data using a FLRW model that the expansion of the universe is accelerating may be misleading. This is important, for it is difficult to explain theoretically the postulated dark energy that causes the acceleration of the universe.

Recently, Barausse, Matarrese and Riotto [6, 7] have perturbed a homogeneous and isotropic FLRW universe to second order, and shown that the luminosity distance-red shift relation implies a non-vanishing cosmic variance of the deceleration parameter implying an intrinsic theoretical uncertainty in its determination. They uncover an infrared long-wavelength effect that depends on the fluctuation spectrum n_s. For $n_s - 1 \ll 0$, a unit variance is obtained in second order perturbation that generates a significant cosmic variance in the deceleration parameter q_0 for cosmological perturbations with the largest wavelengths. Tomita has investigated the interpretation of the luminosity distance and red shift in a local void [8, 9].

We shall not assume that the universe is a local FLRW model. Instead, we use an exact solution for the inhomogeneous perturbation, based on field equations for an inhomogeneous spherically symmetric distribution of matter, embedded in an asymptotic FLRW universe, specializing to the matter dominated Lemaître-Tolman-Bondi (LTB) model [10, 11, 12, 13, 14, 15, 5].

There have been recent reports of statistically significant anomalies in the cosmic microwave background (CMB) compared to the standard big bang model [16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30]. The WMAP data appear to reveal anomalies at the largest angular scales $> 60^0$. The angular two-point correlation function is suppressed at scales larger than 60^0. When related to Fourier space, the vanishing of the two-point correlation function at large scales reveals an alignment of the quadrupole and octopole moments. This could be due to cosmic variance. Oliveira-Costa et al., [26] studied the quadrupole and octopole data and showed that both are planar and aligned with all maxima and minima falling on a great circle on the sky. Similar results have been reported by Schwarz et al. [28] and Weeks [29]. It is not expected that these anomalies in the WMAP data are due to instrumental failure. The CMB temperature fluctuations have been found to be unevenly distributed between the southern and northern hemispheres with a statistical significance $\sim 2 - 3\sigma$. Such effects could be produced by foreground contamination. If, however, they are true cosmic effects, then they would not agree with the standard predictions of inflation theory and a significant fine-tuning of inflationary potentials and parameters would be required to explain the phenomenon [31].

Schwarz et al., have applied a method assigning directions to the ℓ-th multipole using multipole vectors [28]. They found that the method reveals a high statistical significance (99.9%) CL that the observed quadrupole and octupole are not in agreement with a Gaussian random, statistically isotropic sky as predicted generically by inflation. Surprisingly, they discovered a strong correlation with the orientation of the ecliptic plane (solar system and its motion) but no significant correlation with the Milky Way. The motion of the solar system is related to the measured CMB dipole. This effect could be due to a significant systematic error in the WMAP data or that the largest scales of the CMB sky are dominated by a local foreground. It could also be due to cosmic variance and suggest that we are in a special place in the universe at the present time. The cosmic variance for Gaussian random variables is given by

$$\sigma(C_\ell) = \sqrt{\frac{2}{2\ell+1}}. \tag{1}$$

It is a serious limitation for low multipoles that cannot be avoided. If, on the other hand, it is due to a true cosmic effect, or an absorbing or emitting source of microwave radiation in the solar system, then this will have significant consequences for interpreting cosmological data.

Normally, the known Doppler shifts due to the motion of Earth round the Sun ($30\,\mathrm{km}\,s^{-1}$), the motion of the solar system in the Galaxy ($\sim 250\,\mathrm{km}\,s^{-1}$) and the motion of the Galaxy in the Local group of about $600\,\mathrm{km}\,s^{-1}$ in the direction $l \sim 260\,^0, b \sim 30^0$ are removed from the data. Any residual velocities observed would not be expected to produce an alignment of the axes of lower multipoles. An alternative possibility is the existence of inhomogeneities on a scale approaching the scale of the visible universe itself. The CMB data show that such inhomogeneities must have a density contrast less than 10^{-4} over scales of order c/H_0. The deviation of the observer from Hubble flow induced by the perturbations give a variation in temperature of amplitude [32, 33]:

$$\left|\frac{\delta T}{T}\right| \sim \left| q_0 \left(\frac{\delta\rho}{\rho}\right) \frac{a_s}{c/H_0}\right|, \tag{2}$$

where a_s is the scale of the perturbation and q_0 is the deceleration parameter for $t = t_0$ and $z = 0$. The gravitational potential effects produce a fluctuation of order

$$\left|\frac{\delta T}{T}\right| \sim \left(\frac{\delta\Phi}{\Phi}\right)\left(\frac{\delta\rho}{\rho}\right)\left(\frac{a_s}{c/H_0}\right)^2, \tag{3}$$

where Φ is the gravitational potential. The angular scale of the perturbation is of order $a_s H_0/c$ and at the surface of last scattering it is

$$(\delta\rho/\rho)_r = (1+z_r)[a_{sr}(1+z_r)H_r(1+z_r)^{-3/2}]^2, \tag{4}$$

where the subscript r denotes the quantity at the surface of last scattering (recombination).

We have approximately

$$\frac{G\delta M}{R} \sim v\delta v, \tag{5}$$

for a perturbation of excess mass δM with an induced peculiar velocity δv in the Hubble flow $v = H_0 R$, where R is the distance from the center of the density perturbation. This corresponds to a density contrast

$$\frac{\delta\rho}{\rho} \sim \left|\frac{1}{q_0}\frac{\delta v}{H_0}\frac{R^2}{a_s^3}\right|. \tag{6}$$

We shall investigate the interpretation of the red shift and luminosity distance relations in the LTB model. We will also determine the possible lack of complete smoothness of the temperature fluctuations and correlation functions across the sky predicted by the LTB model. The solution can be shown to have a high degree of isotropy and homogeneity at large scales, but there is a generic component of inhomogeneity and anisotropy that is direction dependent and correlated with the location of the origin in spherical polar coordinates.

2 The Lemaître-Tolman-Bondi Model

The CMB temperature effects were first discussed by Sachs and Wolfe [32] and Kantowski [34] considering small density contrasts over large scales for an Einstein-deSitter universe and that the observer is located in a perturbation, or the perturbations were so distant that the radiation passing through them occurred at last scattering. Other situations were discussed by Rees and Sciama [33] and also by Dyer [35]. Raine and Thomas [36] analyzed the situation in which the observer is situated near the edge of a large-scale small amplitude density enhancement in an open universe. We shall investigate in the following, a small amplitude and large scale spherically symmetric density distribution with the observer in a spatially flat universe.

The inhomogeneous line element in comoving coordinates can be written as (see Appendix A):

$$ds^2 = dt^2 - R'^2(t,r)f^{-2}(r)dr^2 - R^2(t,r)d\Omega^2, \tag{7}$$

where we choose units $G = c = 1$, $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ and $f(r)$ is an arbitrary function of r only. For the matter dominated LTB model with zero pressure $p = 0$ and zero cosmological constant $\Lambda = 0$, the Einstein field equations demand that $R(t,r)$ satisfies

$$2R\dot{R}^2 + 2R(1 - f^2) = F(r), \tag{8}$$

with F being an arbitrary function of class C^2, $\dot{R} = \partial R/\partial t$, and $R' = \partial R/\partial r$. There exist three possible solutions depending on whether $f^2 < 1, = 1, > 1$ and they correspond to elliptic (closed), parabolic (flat), and hyperbolic (open) cases, respectively.

The proper density of matter can be expressed as

$$\rho = \frac{F'}{16\pi R' R^2}. \tag{9}$$

By using (8), we can solve (9) to obtain

$$\Omega - 1 \equiv \frac{\rho}{\rho_c} - 1 = \frac{1}{3H_{\rm eff}^2}\biggl(\frac{1-f^2}{R^2} - 2\frac{f}{R}\frac{f'}{R'}\biggr). \tag{10}$$

We have the three possibilities for the curvature of spacetime: 1) $f^2 > 1$ open ($\Omega - 1 < 0$), 2) $f^2 = 1$ flat ($\Omega - 1 = 0$), $f^2 < 1$ closed ($\Omega - 1 > 0$). We define a critical density in analogy with the FLRW model for $f^2 = 1$:

$$8\pi\rho_c = \frac{\dot{R}^2}{R^2} + 2\frac{\dot{R}}{R}\frac{\dot{R}'}{R'}. \tag{11}$$

This corresponds to the critical density for flat spatial sections $t = \text{const}$.

We can define two Hubble parameters $H_r(t,r)$ and $H_\perp(t,r)$ for the local expansion of a spherically symmetric density perturbation, corresponding to the radial and perpendicular directions of expansion, respectively. We have

$$H_r = \frac{\dot{l}_r}{l_r} = \frac{\dot{R}'}{R'}, \quad H_\perp = \frac{\dot{l}_\perp}{l_\perp} = \frac{\dot{R}}{R}, \tag{12}$$

where l denotes the proper distance defined by

$$l_r = R'(t,r) f^{-1} dr, \quad l_\perp = R(t,r) d\Omega. \tag{13}$$

In terms of the effective Hubble parameter

$$H_{\rm eff}^2 = \frac{1}{3}(H_\perp^2 + 2H_\perp H_r), \tag{14}$$

we can obtain the analogy of the Friedmann equation for a spatially flat density perturbation:

$$H_{\rm eff}^2 = \frac{8\pi\rho_c}{3}. \tag{15}$$

The total mass of matter within comoving radius r is

$$M(r) = \frac{1}{4}\int_0^r dr f^{-1} F' = 4\pi \int_0^r dr \rho f^{-1} R' R^2. \tag{16}$$

Since the WMAP data [37, 39] shows that the universe is spatially flat to within a few percent, we shall consider the globally flat case $f^2 = 1$. The metric reduces to

$$ds^2 = dt^2 - R'^2 dr^2 - R^2 d\Omega^2, \tag{17}$$

where

$$R(t,r) = r[t+\beta(r)]^{2/3}, \tag{18}$$

and $\beta(r)$ is an arbitrary function of r of class C^2. The metric (17) becomes

$$ds^2 = dt^2 - (t+\beta)^{4/3}(Y^2 dr^2 + r^2 d\Omega^2), \tag{19}$$

where

$$Y = 1 + \frac{2r\beta'}{3(t+\beta)}, \tag{20}$$

and

$$\rho = \frac{1}{6\pi(t+\beta)^2 Y}. \tag{21}$$

The arbitrary function $\beta(r)$ can be specified in terms of a density on some space-like hypersurface $t = t_0$. The metric and density are singular on the two hypersurfaces $t+\beta = 0$ and $Y = 0$, namely, $t_1 = -\beta$ and $t_2 = -\beta - 2r\beta'/3$, respectively. The model is only valid for $t > \Sigma(r) \equiv \mathrm{Max}[t_1(r), t_2(r)]$, and the hypersurface $t(r) = \Sigma(r)$ defines the big-bang. However, our pressureless model requires that the surface $t(r) = \Sigma(r)$ describes the surface on which the universe becomes matter dominated (in the LFRW model this occurs at $z \sim 10^4$). We observe that even in the spatially flat LTB model, different parts of the universe can enter the matter dominated era at different times.

For $\beta = 0$ and in the limit $t \to \infty$ we obtain the Einstein-deSitter universe

$$ds^2 = dt^2 - a^2(t)(dr^2 + r^2 d\Omega^2). \tag{22}$$

where $a(t) = t^{2/3}$. Thus, for $\beta = 0$ we obtain the FLRW model. Moreover, the expanding flat LTB model necessarily evolves to the homogeneous and isotropic LFRW model for a non-vanishing density, whatever the initial conditions.

3 Paths of Light Rays in the Inhomogeneous Cosmology

The luminosity distance between an observer at the origin of coordinate system $t_0, 0$ and the source at $(t_e, r_e, \theta_e, \phi_e)$ is

$$d_L = \left(\frac{\mathcal{L}}{4\pi\mathcal{F}}\right)^{1/2} = R(t_e, r_e)[1 + z(t_e, r_e)]^2, \tag{23}$$

where $\mathcal{L}$ is the absolute luminosity of the source, $\mathcal{F}$ is the measured flux and $z(t_e, r_e)$ is the red shift (blue shift) for a light ray emitted at (t_e, r_e) and observed at $(t_0, 0)$.

For $\theta = \phi =$ constant we have for $X(r,t) = R(r,t)/f(r)$:

$$\frac{dt}{dr} = \frac{R'(r,t)}{f(r)}. \tag{24}$$

Consider two rays emitted by a source with a small time separation τ. The equation of the first ray is

$$t = T(r), \tag{25}$$

while for the second ray, we have

$$t = T(r) + \tau(r). \tag{26}$$

The equation of a ray and the rate of change of $\tau(r)$ along the path is

$$\frac{dT(r)}{dr} = -\frac{R'}{f}[T(r), r], \tag{27}$$

$$\frac{d\tau(r)}{dr} = -\tau(r)\dot{R}'[T(r), r], \tag{28}$$

where

$$\dot{R}'[T(r), r] = \dot{R}'|_{r,T(r)}. \tag{29}$$

Choosing $\tau(r_e)$ to be the period of a spectral line at r_e, we get

$$\frac{\tau(0)}{\tau(r_e)} = \frac{\nu(r_e)}{\nu(0)} = 1 + z(r_e), \quad z = 0 \quad \text{for} \quad r_e = 0. \tag{30}$$

The red shift considered as a function of r along the light cone is determined by

$$\frac{dz}{dr} = (1 + z)\dot{R}'[T(r), r]. \tag{31}$$

For a light ray travelling from (t_1, r_1) to $(t_0, 0)$ the shift z_1 is

$$\ln(1 + z_1) = -\int_0^{r_1} dr \dot{R}'[T(r), r]. \tag{32}$$

We can compare this to the result obtained from a FLRW universe model by relabelling the radial coordinate $\bar{r} = R[T(r), r]$ and choosing the solution $f = 1$, which corresponds for $p = \Lambda = 0$ to a flat, inhomogeneous matter dominated solution. Moreover, we choose

$$a_1(r) = \dot{R}[T(r), r], \quad a_2(r) = \ddot{R}[T(r), r], ... \tag{33}$$

as functions of r only. By expanding (32), we obtain

$$\ln(1+z_1) = \int_0^{r_1} dr \frac{a_1'}{1-a_1} - \int_0^{r_1} dr \frac{a_1 a_1' - a_2}{1-a_1} = -\ln(1-a_1) - \int_0^{r_1} dr \frac{M'(r)}{(1-a_1)}. \tag{34}$$

Two terms contribute to the cosmological red shift: the contribution due to the expansion of the universe, and the shift due to the difference between the potential energy per unit mass at the source and at the observer. For the FLRW case, $M'(r) = 0$ and there is no gravitational shift. The integral in Eq.(34) for small r_1 can be neglected, and expanding the logarithms on both sides we find for small r_e or small $t_0 - t_e$:

$$z(t_e, r_e) = H_\perp(t_e, r_e) d_L(t_e, r_e). \tag{35}$$

The difference between this result and the one obtained from FLRW is that it is *local* and depends on the angular Hubble parameter $H_\perp = \dot{R}/R$ rather than on the FLRW Hubble parameter $H_{FLRW} = \dot{a}/a$.

A formula for the luminosity distance in an exact FLRW universe is given by [41]:

$$(d_L)_{FLRW}(z) = c(1+z)\int_0^z \frac{du}{H(u)}. \tag{36}$$

This can be reexpressed as

$$(d_L)_{FLRW}(z) = c(1+z)|1-\Omega_0|^{-1/2} H_0^{-1} S\biggl[|1-\Omega_0|^{1/2} H_0 \int_0^z \frac{dz}{H(z)}\biggr], \tag{37}$$

where $S(x) = \sin(x), \Omega_0 > 1, \sinh(x), \Omega_0 < 1, x, \Omega_0 = 1$. For a flat universe

$$(d_L)_{FLRW}(z) = c(1+z)H_0^{-1}\int_0^z du \exp\biggl[-\int_0^u [1+q(v(r))] d\ln(1+v)\biggr]. \tag{38}$$

This result only depends on the assumption of a FLRW universe and does not depend directly on solutions of Einstein's gravitational equations. In our inhomogeneous cosmology, we must generalize the formula for d_L:

$$(d_L)_{LTB}(z(r)) = c(1+z(r))H_{\perp 0}^{-1}\int_0^{z(r)} du \exp\biggl[-\int_0^u [1+q(v,r)] d\ln(1+v)\biggr]. \tag{39}$$

Thus, we interpret the angular Hubble parameter $H_\perp(t,r) = \dot{R}(t,r)/R(t,r)$ as the Hubble parameter that replaces the FLRW expression $H(t) = \dot{a}(t)/a(t)$.

We must now interpret the Hubble parameter and red shift measurements in our model given a void on the scale of ~ 100 Mpc or a large scale perturbation enhancement of the scale of the Hubble horizon $\sim c/H$. Observers at spatially separated locations in the universe would interpret measurements of z, H and d_L differently from one another. We interpret the deceleration parameter in the FLRW model as

$$q_0 = -\frac{\ddot{a}(t_0)}{a(t_0)H_0^2}, \tag{40}$$

and from the SNe Ia supernovae observations arrive at a value $q_0 < 0$ corresponding to an accelerating universe. However, an observer in our LTB model universe in a different, *causally disconnected location in the universe* could observe different q_0 and luminosity distance d_L and arrive at the conclusion that the universe is not accelerating. Thus, averaging all the local observations of d_L and evaluations of q_0 can lead to a cosmic variance on the determination of the deceleration parameter. A significant generic, theoretical uncertainty is generated in the determination of the deceleration parameter q_0.

Luminosity distances are determined by supernovae measurements, which cannot directly determine the instantaneous expansion rate or deceleration rate. Thus, to determine the expansion history, assumptions must be made about the evolution of $H(z)$ and $q(z)$. We find that our inhomogeneous, spherically symmetric model of large scale perturbation enhancements, reaching out to the Hubble horizon scale c/H, can lead to significant differences in the local determinations of H and d_L and an observer independent inference of the rate of expansion and deceleration of the universe has a generic, built-in theoretical uncertainty. If the universe were locally LTB for a matter dominated universe rather than FLRW, then the Hubble parameter based on the observed LTB values of z determined through the relation (38) would be position and red shift dependent. The dependence of H versus z and z versus d_L were given in [14, 15] for different evolving ages of the universe. It was found that $H(z)$ strongly depends on the choice of model for the LFRW case, and the choice of the deceleration parameter q_0 in (38) and (39) through which we interpret the observations. The calculations in [14, 15] were based on the equation describing the ratio of the local density to the density of the LFRW universe taken along the past light cone for a void ($f^2 = 1$):

$$\Omega_{\rm void}[z(r)] \equiv \frac{\rho[T(r),r]}{\rho_{LFRW}[T(r)]} = \frac{3F'(r)[T(r)+\beta_0]^2}{8R'[T(r),r]R^2[T(r),r]}, \tag{41}$$

where β_0 is a constant.

4 CMB Fluctuations and Inhomogeneity

The main assumptions of the standard big-bang model are that the universe is spatially flat and dominated in the matter era by cold dark matter (CDM) and dark energy. The universe enters the matter dominated era with scale invariant, Gaussian, adiabatic initial fluctuations generated in the early universe by a period of inflation. Through a hierarchical gravitational instability process these uniform fluctuations started growing after decoupling (surface of last scattering) to form the large scale structure in the form of galaxies and clusters of galaxies that we see today.

In spatially flat models, the density contrast is define by

$$\delta(\mathbf{x}) \equiv \frac{\delta\rho(\mathbf{x})}{\bar{\rho}} = \frac{\rho(\mathbf{x})-\bar{\rho}}{\bar{\rho}}, \tag{42}$$

where $\bar{\rho}$ denotes the mean density. Its Fourier transform is

$$\delta_k = \frac{1}{V}\int d^3x\delta(\mathbf{x})\exp(i\mathbf{k}\cdot\mathbf{x}). \tag{43}$$

The power spectrum $|\delta_k|^2$ corresponds to statistical random gaussian fluctuations with the rms density fluctuations

$$\frac{\delta\rho}{\rho} = \langle\delta(\mathbf{x})\delta(\mathbf{x})\rangle^{1/2}. \tag{44}$$

For an isotropic power spectrum (i.e. depending on $k = |\mathbf{k}|$ instead of $\mathbf{k}$):

$$\left(\frac{\delta\rho}{\rho}\right) = \frac{1}{V}\int_0^\infty \frac{k^3|\delta_k|^2}{2\pi^2}\frac{dk}{k}. \tag{45}$$

The autocorrelation function is defined by

$$\xi(\mathbf{r}) = \langle\delta(\mathbf{x}+\mathbf{r})\delta(\mathbf{x})\rangle, \tag{46}$$

which is the Fourier transform of the power spectrum $|\delta_k|^2$ and $\xi(0) = (\delta\rho/\rho)^2$.

Since our LTB model is confined to the matter dominated era, it is reasonable to assume that to a first approximation in our inhomogeneous flat model the density perturbations grow like $\delta_+ \equiv (\delta\rho/\rho) \propto t^{2/3}$ independently of the scales. Moreover, we assume that the fluctuations entering the matter dominated era on the surface $t(r) = \Sigma(r)$ have the inflationary scale invariant behavior of the CDM LFRW model. In the LFRW model, we have for the growing mode [15]:

$$\delta_{FLRW}(t) = \delta_{FLRW}(t_{eq})\left(\frac{t_{FLRW}}{t_{eq}}\right)^{2/3}, \tag{47}$$

where t_{eq} and t_{FLRW} denote the time of decoupling and the time from the initial FLRW singularity to a given coordinate value of time t, respectively. In the FLRW model these times are the same everywhere in space.

In the LTB model for a spatially flat universe, we have

$$\delta_{LTB}(t,r) = \delta_{FLRW}(t_\Sigma(r),r)\left[\frac{t_{LTB}(r)}{t_\Sigma(r)}\right]^{2/3}, \tag{48}$$

where now $t_{LTB}(r) = t - \Sigma(r)$ is the time from the initial singularity $t > \Sigma(r)$ to a given coordinate time t. For some time coordinate value t:

$$\delta_{LTB}(t,r) = \delta_{LFRW}(t)\left[\frac{t_{LTB}(r)}{t_{FLRW}}\right]^{2/3}. \tag{49}$$

We have made the simplifying assumption:

$$\delta_{LFRW}(t_{\rm eq}) = \delta_{LTB}(t_\Sigma(r),r) \quad \text{and} \quad t_{\rm eq} = t_\Sigma(r) \quad \text{for all r}. \tag{50}$$

On some spacelike hypersurface t = constant, the gravitational amplification of small primeval perturbations depend on position.

For some spacelike hypersurface t = const, the gravitational amplification of the primeval fluctuation perturbations depends on the spatial position. The larger $t_{LTB}(r)$ for a given r, the more developed structure we expect to see. Also, there can be a different observable distribution of fluctuations in different parts of the sky depending on $t_{LTB}(r)$ and the location of the observer.

We can now write the two-point correlation function as

$$\xi_{LTB}(t_0,r) = \langle\delta_{LTB}(t_0,r)\delta_{LTB}(t_0,0)\rangle. \tag{51}$$

The correction to the LFRW correlation function ξ_{FLRW} is given by

$$\xi_{LTB}(r) = \xi_{FLRW}(r)\Big[\frac{t_{LTB}(r)}{t_F}\Big]^{2/3}. \tag{52}$$

The CMB temperature anisotropies generated by scalar perturbations of the LFRW spacetime metric can be written as

$$ds^2 = a^2(\eta)[(1+2\Phi)d\eta^2 - (1-2\Phi)d\vec{x}^2], \tag{53}$$

where $a(\eta)$ is the LFRW scale factor and η is the conformal time. At the last scattering surface hypersurface, the anisotropy contributes $\Phi/3$ sources. For a non-zero $d\Phi/d\eta$ an additional integral contribution arises. for a scale free primordial spectrum $|\Phi_k|^2 = |A^2/k^3|$ the CMB angular power spectrum amplitude can be written [31]:

$$\langle\Big(\frac{\delta T}{T}\Big)^2\rangle_\ell = 36\pi^2\Big(\frac{2\ell+1}{\ell(\ell+1)}\Big)A^2K_\ell^2, \tag{54}$$

where

$$K_\ell^2 = 2\ell(\ell+1)\int_0^\infty\frac{dk}{k}[j_\ell(k\eta_0) + 6\int_{\eta_r}^{\eta_0}d\eta\frac{df_k}{d\eta}j_\ell(k(\eta_0-\eta))]^2, \tag{55}$$

and $df_k/d\eta$ denotes $d\Phi_k/d\eta$ normalized to $A/k^{3/2}$. A flat CDM model has $d\Phi/d\eta$ and $K_\ell^2 = 1$, while a flat ΛCDM has $K_\ell^2 > 1$ with an amplification exceeding the Sachs-Wolfe plateau.

For the late time universe, it was argued by Contaldi et al. [31] that the suppression of the large scale (low ℓ moments) power spectrum could be explained by fine-tuning the coefficients K_ℓ^2 to be small with certain solutions of late time Φ. On the other hand, the amplitude of metric perturbations at horizon crossing in inflationary models could fine-tune $k^{3/2}\Phi_k$ to give a small contribution at large angular scales. This would involve fine-tuning the shapes of inflaton potentials in an ad hoc way.

In our LTB model, the growth of the angular power spectrum will have the form

$$\langle\Big(\frac{\delta T}{T}\Big)\rangle_\ell^{LTB}(t,r) = \langle\Big(\frac{\delta T}{T}\Big)\rangle_\ell^{FLRW}(t)\Big[\frac{t_{LTB}(r)}{t_\Sigma}\Big]^{2/3}. \tag{56}$$

We see that there will be a difference in the observed CMB temperature fluctuations in different parts of the universe, depending on the value of r and the location of the observer.

5 Isotropy of the Cosmic Microwave Background

The standard description of cosmology is based on the LFRW model and the cosmological principle – the statement that the universe is the same in all directions and locations. The assumption of isotropy necessarily implies that the statistical properties of the CMB should be the same in all directions on the sky. The FIRAS experiment on the COBE satellite [38, 39] demonstrated that the mean temperature of the CMB is isotropic to a high precision. However, the sensitivity of the WMAP data [37] can be used to test the isotropy of the angular fluctuations in the CMB. Hansen, Banday and Górski [16] have studied the WMAP power spectrum extracted from the CMB using patches at different directions on the sky. In the lower angular range $\ell \sim 2 - 40$, they find preliminary indications for non-zero differences in the power spectrum in the northern and southern hemispheres oriented along the galactic colatitude and longitude $(80^0, 57^0)$, close to the ecliptic pole. Eriksen et al. [17] and Park [18] were the first to discover a difference between the northern and southern hemispheres in non-gaussianity tests. Similar results have been found by Coles et al. [19], Cruz et al. [20], Vielva et al. [21], Komatsu, Spergel and Wandelt [22], Larson and Wandelt [23], Land and Magueijo [25], Oliveira et al. [26]. An analysis of the variation in cosmological parameters associated with the acoustical peak in the power spectrum was carried out by Donoghue and Donoghue, to see whether there exists a correlation between the height and location of the peak [30]. A measurable dipole effect in the CMB would indicate that there exists a spatial asymmetry. However, at present after the subtraction of the dipole contribution from the WMAP data, there is no indication of a statistically significant dipole contribution.

The observed non-zero differences in the power spectrum in the northern and southern hemispheres can be ascribed in our LTB model as being due to a difference in $t_{LTB}(r)$ in the locations of the northern and southern hemispheres, which will lead to a correction factor described in Eqs.(49) and (56):

$$\Delta = \left(\frac{t_{LTB}(r_s)}{t_\Sigma}\right) - \left(\frac{t_{LTB}(r_n)}{t_\Sigma}\right), \tag{57}$$

where r_s and r_n denote the position locations of the northern and southern hemispheres.

The claims about the detection of an asymmetric distribution of large scale power in the CMB anisotropy as measured by the WMAP satellite are found at large angular scales. For scales $\ell > 40$ (corresponding to an angular scale $\sim 3^0$), the distribution of the CMB fluctuations is consistent with the hypothesis of isotropy and homogeneity, although there have been indications of possible foreground contamination around the first peak $\ell \sim 220$. However, for lower ℓ multipole values, a strong difference between the northern and southern hemispheres (for galactic and ecliptic frames of reference) was found. The CMB power spectrum at large scales $(\ell = 2 - 40)$ is found to be significantly much lower in the northern hemisphere than in the southern hemisphere, which leads to a different estimate of the cosmological parameters in these hemispheres.

When attention is focussed on three parameters to which the analysis of the anisotropy and non-Gaussianity is most sensitive, namely, the spectral index n_s, the amplitude of fluctuations A and the optical depth τ, the preferred value for the optical depth in the north is $\tau < 0.08$, whereas in the south it is $\tau = 0.24^{+0.06}_{-0.07}$ (68% confidence level). The latter result is inconsistent with $\tau = 0$ at the 2σ level. The WMAP collaboration estimate $\tau = 0.17$ could thus originate in structure associated with the southern hemisphere. By setting a prior on τ, values of the spectral index n_s are found that are inconsistent between the opposite hemispheres.

Let us now assume that a mechanism is operative such that the amplitude (56) is suppressed for large angular scales in the sky for low multipoles. The observer is then able to see small dipole, quadrupole and octopole contributions generated by an inhomogeneous perturbation enhancement at scales c/H and the gravitational potential at the location of the enhancement. An observer at (r_0, t_0) measures a temperature $T = T_0$. At the time of recombination $z \sim 10^3$, the temperature is $T_r \sim 10^3 T_0$. The red shift varies somewhat with angle θ between the direction of a light ray emitted at (r_0, t_0) and absorbed at (r_r, t_r) and a light ray directed toward the center of the density perturbation. The apparent temperature of the CMB as observed in the direction θ is given by

$$T_b(\theta) = \frac{T_r}{1 + z(\theta)} = T_{\rm av}\frac{1 + z_{\rm av}}{1 + z(\theta)}, \tag{58}$$

where $T_{\rm av}$ and $z_{\rm av}$ are the temperature and red shift averaged over the whole sky and we have $T_{\rm av} \sim T_0$.

The dipole, quadrupole and octopole moments, $\mathcal{D}$, $\mathcal{Q}$ and $\mathcal{O}$ are defined by [40]

$$\mathcal{D} \equiv \frac{1}{T_{\rm av}}\int_0^\pi T_b(\theta) Y_{10}(\theta)\sin\theta d\theta, \tag{59}$$

$$\mathcal{Q} \equiv \frac{1}{T_{\rm av}}\int_0^\pi T_b(\theta) Y_{20}(\theta)\sin\theta d\theta, \tag{60}$$

$$\mathcal{O} \equiv \frac{1}{T_{\rm av}}\int_0^\pi T_b(\theta) Y_{30}(\theta)\sin\theta d\theta, \tag{61}$$

where e.g. $Y_{10} = \sqrt{3/4\pi}\cos\theta$. An observer looking toward the center of the spherically symmetric LTB density perturbation will see an axially symmetric distribution. The dipole $\mathcal{D}$, quadrupole $\mathcal{Q}$ and the octopole $\mathcal{O}$ will be aligned in this plane, with the angle θ defining the trajectory of a light ray arriving to the observer located at (r_0, t_0). The dipole, quadrupole and octopole contributions are produced by gradients of the gravitational potential associated with the large scale perturbation enhancement. The dipole contribution is small and undetected in the WMAP data, while the quadrupole and octopole contributions have been found to be planar and the planes are aligned to a statistically anomalous degree [26, 28, 29].

A possible explanation for the quadrupole and octopole anomaly is that the cosmic topology has the form of a toroidal universe with one small dimension of order one-half the horizontal scale, in the direction towards Virgo (small universe model). Another possibility is that the universe takes on the topological shape of a dodecahedron [42]. These possible explanations appear to have been ruled out by Cornish and collaborators [43].

6 Conclusions

We have used a cosmological model with the exact inhomogeneous LTB solution for a matter dominated, large scale perturbation enhancement at Hubble horizon scale c/H with $p = \Lambda = 0$ to demonstrate that observers located in different places in the universe can differ significantly in their determinations of the evolution of red shift and Hubble expansion rate. This will significantly influence the theoretical interpretation of the locally determined values of $H(z)$ and $d_L(z)$. Averaging over all the observers results will yield a generic cosmic variance in the determination of the deceleration parameter. This in turn will lead to a variance in the conclusion as to whether the universe is undergoing an accelerating phase. An independent way to determine luminosity distances to supernova and distance objects in the universe could settle the theoretical ambiguity.

The strong difference between the distribution of CMB fluctuations in the northern and southern hemispheres discovered by analysis of the WMAP power spectrum data for large scales with multipole $\ell = 2 - 40$ values, in the ecliptic and galactic frames of reference, can be explained as a cosmological effect by a large scale inhomogeneous perturbation enhancement, described by our exact inhomogeneous solution of Einstein's field equations. The time evolution of the universe as measured from the surface of last scattering in the matter dominated LTB model can produce a correction factor $(t_{LTB}(r)/t_\Sigma)^{2/3}$ for the fluctuations that can differ significantly between the northern and southern hemispheres. However, the uneven distribution of fluctuations could be due to systematic effect in the WMAP data or be due to foreground contamination. Hopefully, the WMAP2 analysis of the CMB data will decide whether or not the non-Gaussian non-isotropic effect has a cosmological origin or is due to some other non-cosmological mechanism.

The possible statistically significant alignment of the quadrupole and octopole moments in the WMAP data can be explained, in our large scale inhomogeneous model, by an off-center observer seeing an axisymmetric alignment of the quadrupole and octopole moments as the observer receives light rays from the center of the large scale perturbation enhancement.

7 Appendix A

For the sake of notational clarity, we write the FLRW line element

$$ds^2 = dt^2 - a^2(t)\Big(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2\Big), \tag{62}$$

where $d\Omega^2 = d\theta^2 + \sin\theta^2 d\phi^2$.

Let us now consider the more general spherically symmetric inhomogeneous line element [10, 11, 12]:

$$ds^2 = dt^2 - X^2(r,t)dr^2 - R^2(r,t)d\Omega^2. \tag{63}$$

The energy-momentum tensor ${T^\mu}_\nu$ takes the form

$$ {T^\mu}_\nu = (\rho + p)u^\mu u_\nu - p{\delta^\mu}_\nu, \tag{64} $$

where $u^\mu = dx^\mu/ds$ and, in general, the density $\rho = \rho(r,t)$ and the pressure $p = p(r,t)$ depend on both r and t. We have for comoving coordinates $u^0 = 1, u^i = 0, \quad (i = 1,2,3)$. Moreover, the total mass is

$$ M(r) = 4\pi \int_0^r T\sqrt{-g} = 4\pi \int_0^r dr \rho(r) X R^2, \tag{65} $$

where $g = \mathrm{Det}(g_{\mu\nu}) = -X^2 Y^4 \sin^2\theta$. From this follows that

$$ M' = \frac{dM}{dr} = 4\pi \rho X R^2. \tag{66} $$

The Christoffel symbols are

$$ {\Gamma^\sigma}_{\mu\nu} = \frac{1}{2} g^{\sigma\alpha}(\partial_\mu g_{\nu\alpha} + \partial_\nu g_{\mu\alpha} - \partial_\alpha g_{\mu\nu}). \tag{67} $$

The non-vanishing Christoffel symbols are

$$ {\Gamma^1}_{11} = \frac{X'}{X}, \quad {\Gamma^0}_{11} = X\dot{X}, \quad {\Gamma^1}_{01} = \frac{\dot{X}}{X}, \quad {\Gamma^2}_{02} = {\Gamma^3}_{03} = R\dot{R}\sin^2\theta, $$

$$ {\Gamma^0}_{22} = R\dot{R}, \quad {\Gamma^1}_{33} = -\frac{RR'}{X^2}\sin^2\theta, \quad {\Gamma^2}_{33} = -\sin\theta\cos\theta, \quad {\Gamma^0}_{23} = \cot\theta, \tag{68} $$

where $X' = \partial X/\partial r$ and $\dot{X} = \partial X/\partial t$. The Einstein gravitational equations are

$$ G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = -8\pi T_{\mu\nu}, \tag{69} $$

where $R = g^{\mu\nu} R_{\mu\nu}$ and Λ is the cosmological constant. We have

$$ {G_0}^0 \equiv 2\frac{\dot{X}\dot{R}}{XR} + \frac{1+\dot{R}^2}{R^2} - \frac{1}{X^2}\Big(2\frac{R''}{R} + \frac{R'^2}{R^2} - 2\frac{X'R'}{XR}\Big) = 8\pi {T_0}^0 + \Lambda = 8\pi\rho + \Lambda, \tag{70} $$

$$ {G_1}^1 \equiv 2\frac{\ddot{R}}{R} + \frac{1+\dot{R}^2}{R^2} - \frac{R'^2}{X^2R^2} = 8\pi {T_1}^1 + \Lambda, \tag{71} $$

$$ {G_2}^2 \equiv \frac{\ddot{X}}{X} + \frac{\ddot{R}}{R} + \frac{\dot{X}\dot{R}}{XR} - \frac{1}{X^2}\Big(\frac{R''}{R} - \frac{X'R'}{XR}\Big) = 8\pi {T_2}^2 + \Lambda = 8\pi {T_3}^3 + \Lambda = -8\pi p + \Lambda, \tag{72} $$

$$ {G_1}^0 \equiv -2\Big(\frac{\dot{R}'}{R} - \frac{\dot{X}R'}{XR}\Big) = 8\pi {T_1}^0 = -8\pi {T_0}^1 = 0. \tag{73} $$

From Eq. (73), we find that

$$ X(r,t) = \frac{1}{f(r)} R'(r,t). \tag{74} $$

For an isotropic pressure ${T_1}^1 = {T_2}^2 = {T_3}^3$ and the pressure $p = p(t)$ only depends on the time t. We now obtain the two equations

$$\frac{\dot{R}^2}{R^2} + 2\frac{\dot{R}'}{R'}\frac{\dot{R}}{R} + \frac{1}{R^2} - \frac{f^2}{R'^2}\left(\frac{2R''}{R} + \frac{R'^2}{R^2} - 2\frac{\left(\frac{R'}{f}\right)' f}{R}\right) = 8\pi\rho + \Lambda, \tag{75}$$

$$\ddot{R} + \frac{1}{3}\frac{\dot{R}^2}{R} - \frac{1}{3}\frac{\dot{R}'\dot{R}}{R'} + \frac{1}{3}\frac{1}{R} - \frac{2}{3}\frac{f^2}{R} - \frac{1}{3}\frac{R''f^2}{R'^2} + \frac{1}{3}\frac{\left(\frac{R'}{f}\right)' f^3}{R'^2} = -\frac{4\pi}{3}(\rho + 3p)R + \Lambda R. \tag{76}$$

We now find that for $f = 1$ we obtain

$$\frac{\dot{R}^2}{R^2} + 2\frac{\dot{R}}{R}\frac{\dot{R}'}{R'} = 8\pi\rho + \Lambda, \tag{77}$$

,

$$\ddot{R} + \frac{1}{3}\frac{\dot{R}^2}{R} - \frac{1}{3}\left(\frac{\dot{R}'}{\dot{R}}R'\right) = -\frac{4\pi}{3}(\rho + 3p)R + \Lambda R. \tag{78}$$

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References

[1] S. Perlmutter et al. Ap. J. **517**, 565 (1999); A. G. Riess, et al. Astron. J. **116**, 1009 (1998); P. M. Garnavich, et al. Ap. J. **509**, 74 (1998).

[2] A. G. Riess, et al. astro-ph/0402512.

[3] P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. **75**, 559 (2003), astro-ph/0207347.

[4] N. Straumann, astro-ph/0203330.

[5] For a review of inhomogeneous cosmology and references, see: A Krasiński, *Inhomogeneous Cosmological Models*, Cambridge University Press, 1997.

[6] E. Barausse, S. Matarrese and A. Riotto, astro-ph/0501152.

[7] T. Pyne and M. Birkinshaw, Mon. Not. Roy. Astron. Soc. **348**, 581 (2004), astro-ph/0310841.

[8] K. Tomita, Mon. Not. Roy. Astron. Soc. **326**, 287 (2001), astro-ph/0011484; astro-ph/0011484; Prog. Theor. Phys. **106**, No. 5 (2001), astro-ph/0203125; Prog. Theor. Phys. **108**, 103 (2002), astro-ph/0203125; Ap. J. **584**, 580 (2003), astro-ph/0211137.

[9] H. Mathis et al. Mon. Not. Roy. Astron. Soc. **350**, 287 (2004), astro-ph/0303579.

[10] G. Lemaître, Mon. Not. Roy. Astron. Soc. **91**, 490 (1931); Ann. Soc. Sci. Bruxelles, **A53**, 51 (1933).

[11] R. C. Tolman, Proc. Nat. Acad. Sci. **20**, 169 (1934).

[12] H. Bondi, Mon. Not. Roy. Astron. Soc. **107**, 410 (1947).

[13] W. B. Bonnor, Mon. Not. Roy. Astron. Soc. **167**, 55 (1974).

[14] J. W. Moffat and D. C. Tatarski, Ap. J. **453**, 17 (1990).

[15] J. W. Moffat and D. C. Tatarski, Phys. Rev. **D45**, 3512 (1992).

[16] F. K. Hansen A. J. Balbay and K. M. Górski, astro-ph/0404206; F. K. Hansen, astro-ph/0406393.

[17] H. K. Eriksen et al., Ap. J. **605**, 14 (2004), astro-ph/0307507.

[18] C. G. Park, Mon. Not. Roy. Astron. Soc. **349**, 313 (2004), astro-ph/0307469.

[19] P. Coles et al., astro-ph/0310252.

[20] M. Cruz et al., astro-ph/0405341.

[21] P. Vielva et al., astro-ph/0310273.

[22] E. Komatsu, D. N. Spergel and B. D. Wandelt, astro-ph/ 0302223.

[23] D. L. Larson and B. D. Wandelt, astro-ph/0404037.

[24] P. Mukherjee and Y. Wang, astro-ph/0402602.

[25] K. Land and J. Magueijo, astro-ph/0405519; astro-ph/0407081.

[26] A. de Oliveira-Costa et al. Phys. Rev. **D69**, 063516, astro-ph/0307282.

[27] C. J. Copi, D. Huterer and G. D. Starkman, astro/ph/0310252.

[28] D. J. Schwarz et al. Phys. Rev. Lett. **93**, 221301 (2004), astro-ph/0403353.

[29] J. R. Weeks, astro-ph/0412231.

[30] E. P. Donoghue and J. F. Donoghue, astro-ph/0411237.

[31] C. R. Contaldi, et al. JCAP 0307 (2003) 002, astro-ph/0303636.

[32] R. K. Sachs and A. M. Wolfe, Ap. J. **147**, 73 (1967).

[33] M. J. Rees and D. W. Sciama, Nature, **217**, 511 (1968).

[34] R. Kantowski, Ap. J. **155**, 89 (1969).

[35] C. C. Dyer, Mon. Not. Roy. Astron. Soc. **175**, 429 (1976).

[36] D. J. Raine and E. G. Thomas, Mon. Not. Roy. Astron. Soc. **195**, 649 (1981).

[37] C. L. Bennett et al., Ap. J. Suppl. **148**, 1 (2003), astro-ph/0302207; D. N. Spergel et al., Ap. J. Suppl. **148**, 175 (2003), astro-ph/0302209.

[38] G. F. Smoot et al., Ap. J. Letters **396**, L1 (1996).

[39] C. L. Bennett et al., Ap. J. Letters **464**, L1 (1996).

[40] B. Paczyński and T. Piran, Ap. J. 364, 341 (1990).

[41] M. S. Turner and A. G. Riess, Ap. J. **569**, 18 (2002), astro-ph/0106051.

[42] J. P. Luminet et al., Nature (London) **425**, 593 (2003).

[43] N. J. Cornish, D. N. Spergel, G. D. Starkman and E. Komatsu, astro-ph/0310233.