Multiverses and Cosmology: Philosophical Issues

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January 19, 2006

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Abstract

The idea of a multiverse – an ensemble of universes or universe domains – has received increasing attention in cosmology, both as the outcome of the originating process that generated our own universe, and as an explanation for why our universe appears to be fine-tuned for life and consciousness. Here we review how multiverses should be defined, stressing the distinction between the collection of all possible universes and ensembles of really existing universes, which distinction is essential for anthropic arguments. We show that such realised multiverses are by no means unique, and in general require the existence of a well-defined and physically motivated distribution function on the space of all possible universes. Furthermore, a proper measure on these spaces is also needed, so that probabilities can be calculated. We then discuss several other physical and philosophical problems arising in the context of ensembles of universes, including realized infinities and the issue of fine-tuning – whether very special or generic primordial conditions are more fundamental in cosmology. Then we briefly summarise scenarios like chaotic inflation, which suggest how ensembles of universe domains may be generated, and point out that the regularities underlying any systematic description of truly disjoint multiverses must imply some kind of common generating mechanism, whose testability is problematic. Finally, we discuss the issue of testability, which underlies the question of whether multiverse proposals are really scientific propositions rather than metaphysical proposals.

Key Words: Cosmology; inflation; multiverses; anthropic principle

1 Introduction

Over the past twenty years the proposal of a really existing ensemble of universes – a ‘multiverse’ – has gained prominence in cosmology, even though there is so far only inadequate theoretical and observational support for its existence. The popularity of this proposal can be traced to two factors. The first is that quite a few promising programs of research in quantum and very early universe cosmology suggest that the very processes which could have brought our universe or region of the universe into existence from a primordial quantum configuration, would have generated many other universes or universe regions as well. This was first modelled in a specific way by Vilenkin (1983) and was developed by Linde (Linde 1983, 1990) in his chaotic cosmology scenario. Since then many others, e. g. Leslie (1996), Weinberg (2000), Sciama (1993), Deutsch (1998), Tegmark (1998, 2003), Smolin (1999), Lewis (2000), Weinberg (2000), and Rees (2001) have discussed ways in which an ensemble of universes or universe domains might originate physically. More recently specific impetus has been given to this possibility by superstring theory. It is now claimed by some that versions of these theories provide "landscapes" populated by a large number of vacua, * each of which could occur in or initiate a separate universe domain,* with different values of the physical parameters, such as the cosmological constant, the masses of the elementary particles and the strengths of their interactions (Kachru, et al. 2003; Susskind 2003, 2005, and references therein).

So far, none of these proposals has been developed to the point of actually describing such ensembles of universes in detail, nor has it been demonstrated that a generic well-defined ensemble will admit life. Some writers tend to imply that there is only one possible multiverse, characterised by "all that can exist does exist" (Lewis 2000, see also Gardner 2003). This vague prescription actually allows a vast variety of different realisations with differing properties, leading to major problems in the definition of the ensembles and in averaging, due to the lack of a well-defined measure and the infinite character of the ensemble itself. Furthermore, it is not at all clear that we shall ever be able to accurately delineate the class of all possible universes.

The second factor stimulating the popularity of multiverses is that it is the only scientific way of avoiding the fine-tuning seemingly required for our universe. This applies firstly to the cosmological constant, which seems to be fine-tuned by 120 orders of magnitude relative to what is expected on the basis of quantum field theory (Weinberg 2000, Susskind 2005). If (almost) all values of the cosmological constant occur in a multiverse, then we can plausibly live in one with the very low observed non-zero value; indeed such a low value is required in order that galaxies, stars, and planets exist and provide us with a suitable habitat for life.

This is an example of the second motivation, namely the 'anthropic principle' connection: If any of a large number of parameters which characterize our universe – including fundamental constants, the cosmological constant, and and initial conditions – were slightly different, our universe would not be suitable for complexity or life. What explains the precise adjustment of these parameters so that microscopic and macroscopic complexity and life eventually emerged? One can introduce a "Creator" who intentionally sets their values to assure the eventual development of complexity. But this move takes us beyond science. The existence of a large collection of universes, which represents the full range of possible parameter values, though not providing an ultimate explanation, would provide a scientifically accessible way of avoiding the need for such fine-tuning. If physical cosmogonic processes naturally produced such a variety of universes, one of which was ours, then the puzzle of fine- tuning is solved. We simply find ourselves in one in which all the many conditions for life have been fulfilled.

Of course, through cosmology we must then discover and describe the process by which that collection of diverse universes, or universe domains, was generated, or at least could have been generated, with the full range of characteristics they possess. This may be possible. It is analogous to the way in which we look upon the special character of our Solar System. We do not agonize how initial conditions for the Earth and Sun were specially set so that life would eventually emerge – though at some level that is still a mystery. We simply realize that our Solar System in one of hundreds of billions of others in the Milky Way, and accept that, though the probability that any one of them is bio-friendly is very low, at least a few of them will naturally be so. We have emerged as observers in one of those. No direct fine-tuning is required, provided we take for granted both the nature of the laws of physics and the specific initial conditions in the universe. The physical processes of stellar formation throughout our galaxy naturally leads to the generation of the full range of possible stellar systems and planets.

Before going on, it is necessary to clarify our terminology. Some refer to the separate expanding universe regions in chaotic inflation as 'universes', even though they have a common causal origin and are all part of the same single spacetime. In our view (as 'uni' means 'one') *the Universe* is by definition the one unique connected[1] existing spacetime of which our observed expanding cosmological domain is a part. We will refer to situations such as in chaotic inflation as a *Multi-Domain Universe*, as opposed to a completely causally disconnected *Multiverse*. Throughout this paper, when our discussion pertains equally well to disjoint collections of universes (multiverses in the strict sense) and to the different domains of a Multi-Domain Universe, we shall for simplicity simply use the word "*ensemble*". When the universes of an ensemble are all sub-regions of a larger connected spacetime - the "Universe as a whole"- we have the multi-domain situation, which should be described as such. Then we can reserve "multiverse" for the collection of genuinely disconnected "universes" – those which are not locally causally related.

In this article, we shall critically examine the concept of an ensemble of universes or universe domains, from both physical and philosophical points of view, reviewing how they are to be defined physically and mathematically in cosmology (Ellis, Kirchner and Stoeger 2003, hereafter referred to as EKS), how their existence could conceivably be validated scientifically, and focusing

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[1] "Connected" implies "Locally causally connected", that is all universe domains are connected by C^0 timelike lines which allow any number of reversals in their direction of time, as in Feynman's approach to electrodynamics. Thus, it is a union of regions that are causally connected to each other, and transcends particle and event horizons; for example all points in de Sitter space time are connected to each other by such lines.

upon some of the key philosophical problems associated with them. We have already addressed the physics and cosmology of such ensembles in a previous paper (EKS), along with some limited discussion of philosophical issues. Here we shall summarize the principal conclusions of that paper and then discuss in detail the more philosophical issues.

First of all, we review the description of the the set of possible universes and sets of realised (i. e., really existing) universes and the relationship between these two kinds of sets. It is fundamental to have a general provisionally adequate scheme to describe the set of all possible universes. Using this we can then move forward to describe potential sets of actually existing universes by defining distribution functions (discrete or continuous) on the space of possible universes. A given distribution function indicates which of the theoretically possible universes have been actualized to give us a really existing ensemble of universes or universe domains. It is obviously crucial to maintain the distinction between the set of all possible universes, and the set of all existing universes. For it is the set of all existing universes which needs to be explained by cosmology and physics – that is, by a primordial originating process or processes. Furthermore, it is only an *actually existing* ensemble of universes with the required range of properties which can provide an explanation for the existence of our bio-friendly universe without fine-tuning (see also McMullin 1993, p. 371). A conceptually possible ensemble is not sufficient for this purpose – one needs universes which actually exist, along with mechanisms which generate their existence. We consider in some depth how the existence of such an actually existing ensemble might be probed experimentally and observationally - this is the key issue determining whether the proposal is truly a scientific one or not.

Though the ensemble of all possible universes is undoubtedly infinite, having an infinite ensemble of actually existing universes is problematic – and furthermore blocks our ability to assign statistical measures to it, as we shall discuss in some detail later. For all these reasons, any adequate cosmological account of the origin of our universe as one of a collection of many universes – or even as a single realised universe – must include a process whereby the realised ensemble is selected from the space of all possible universes and physically generated. But it must also provide some metaphysical view on the origin of the set of possible universes as a subset of the set of conceivable universes - which is itself a very difficult set to define[2].

2 Describing Ensembles: Possibility

To characterize an ensemble of existing universes, we first need to develop adequate methods for describing the class of all possible universes. This itself is philosophically controversial, as it depends very much on what we regard as "possible." At the very least, describing the class of all possible universes requires us to specify, at least in principle, all the ways in which universes can

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[2] Science fiction and fantasy provide a rich treasury of conceivable universes, many of which will not be "possible universes" as outlined above.

be different from one another, in terms of their physics, chemistry, biology, etc. We have done this in EKS, which we shall review here.

2.1 The Set of Possible Universes

Ensembles of universes, or multiverses, are most easily represented classically by the structure and the dynamics of a space $\mathcal{M}$ of all possible universes, each of which can be described in terms of a set of states s in a state space $\mathcal{S}$ (EKS). Each universe m in $\mathcal{M}$ will be characterised by a set $\mathcal{P}$ of distinguishing parameters p, which are coordinates on $\mathcal{S}$ (EKS). Each m will evolve from its initial state to some final state according to the operative dynamics, with some or all of its parameters varying as it does so. The course of this evolution of states will be represented by a path in the state space $\mathcal{S}$. Thus, each such path (in degenerate cases, a point) is a representation of one of the universes m in $\mathcal{M}$. The parameter space $\mathcal{P}$ has dimension N which is the dimension of the space of models $\mathcal{M}$; the space of states $\mathcal{S}$ has $N+1$ dimensions, the extra dimension indicating the change of each model's states with time, characterised by an extra parameter, e.g., the Hubble parameter H which does not distinguish between models but rather determines what is the state of dynamical evolution of each model. Note that N may be infinite, and indeed will be so unless we consider only geometrically highly restricted sets of universes.

This classical, non-quantum-cosmological formulation of the set of all possible universes is obviously provisional and not fundamental. Much less should it provide the basis for adjudicating the ontology of these ensembles and their components.[3] It provides us with a preliminary systematic framework, consistent with our present limited understanding of cosmology, within which to begin studying ensembles of universes and universe domains. It is becoming very clear that, from what we are beginning to learn from quantum cosmology, a more fundamental framework will have to be developed that takes seriously quantum issues such as entanglement. Additionally, there are serious unresolved problems concerning time in quantum cosmology. Already at the level of general relativity itself, as everyone recognizes, time loses its fundamental, distinct character. What is given is space-time, not space and time. Time is now intrinsic to a given universe domain and its dynamics and there is no preferred or unique way of defining it (Isham 1988, 1993; Smolin 1991; Barbour 1994; Rovelli 2004; and references therein). When we go to quantum gravity and quantum cosmology, time, while remaining intrinsic, recedes further in prominence and even seems to disappear. The Wheeler-de Witt equation for "wave function of the universe," for example, does not explicitly involve time – it is a time-independent equation. However, our provisional classical formulation receives support from the fact that dynamics and an intrinsic time appear to emerge from it as the universe expands out of the Planck era (see, for instance, Isham 1988 and Rovelli 2004, especially pp. 296-301). Furthermore, as yet there is

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[3] We thank and acknowledge the contribution of an anonymous referee who has pointed this out to us, and has stimulated this brief discussion of the important role of quantum cosmology in defining multiverses.

no adequate quantum gravity theory nor quantum cosmological resolution to this issue of the origin and the fundamental character of time – just tantalizing pieces of a much larger picture. The only viable approach at present is to proceed on the basis of the emergent classical description.

And then there are related issues connected with decoherence – how is the transition from "the wave function of the universe" to the classical universe, or an ensemble of universe domains, effected, and what emerges in this transition? What is crucial here is that as the wave function decoheres an entire ensemble of universes or universe domains may emerge. These would all be entangled with one another. This would provide the fundamental basis for the quantum ontology of the ensemble.[4] Furthermore, it would provide a fundamental connection among a large number of the members of our classically defined $\mathcal{M}$ above. We have already stressed the difference between a multi-domain universe and a true multiverse. An entangled ensemble of universe domains decohering from a cosmological wave function would be an important example of that case. This process of cosmological decoherence, which we as yet do not understand and have not adequately modelled , may turn out to be a key generating mechanism for a really existing multiverse. In that case we would want to define a much more fundamental space of all possible cosmological wave functions. Each of these would generate an ensemble of classical universes or universe domains which we have represented individually in $\mathcal{M}$. We could then map the wave-functions in that more fundamental space into the m of $\mathcal{M}$. As yet, however, we do not have even a minimally reliable quantum cosmology that would enable us to implement that.

Despite our lack of understanding at the quantum cosmological level, and the less than fundamental character of our space $\mathcal{M}$, it enables us proceed with our discussion of cosmological ensembles at the non-quantum level - which is what cosmological observations relate to. While doing so, we must keep the quantum cosmological perspective in mind. Though we are without the resources to elaborate it more fully, it provides a valuable context within which to interpret, evaluate and critique our more modest classical discussion here.

Returning to our description of the space $\mathcal{M}$ of possible universes m, we must recognize that it is based on an assumed set of laws of behaviour, either laws of physics or meta-laws that determine the laws of physics, which all m have in common. Without this, we have no basis for defining it. Its overall characterisation must therefore incorporate a description both of the geometry of the allowed universes and of the physics of matter. Thus the set of parameters $\mathcal{P}$ will include both geometric and physical parameters.

Among the important subsets of the space $\mathcal{M}$ are (EKS): $\mathcal{M}_{\mathrm{FLRW}}$, the subset of all possible Friedmann-Lemaître-Robertson-Walker (FLRW) universes, which are exactly isotropic and spatially homogeneous; $\mathcal{M}_{\mathrm{almost-FLRW}}$, the subset of all universes which deviate from exact FLRW models by only small, linearly growing anisotropies and inhomogeneities; $\mathcal{M}_{\mathrm{anthropic}}$, the subset of all possible universes in which life emerges at some stage in their evolution. This subset

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[4] Again, we thank the same referee for emphasizing the importance of the possibility.

intersects $\mathcal{M}_{\mathrm{almost-FLRW}}$, and may even be a subset of $\mathcal{M}_{\mathrm{almost-FLRW}}$, but does not intersect $\mathcal{M}_{\mathrm{FLRW}}$, since realistic models of a life-bearing universe like ours cannot be exactly FLRW, for then there is no structure.

If $\mathcal{M}$ truly represents all possibilities, as we have already emphasized, one must have a description that is wide enough to encompass *all* possibilities. It is here that major issues arise: how do we decide what all the possibilities are? What are the limits of possibility? What classifications of possibility are to be included? From these considerations we have the first key issue (EKS):

Issue 1: What determines $\mathcal{M}$? Where does this structure come from? What is the meta-cause, or ground, that delimits this set of possibilities? Why is there a uniform structure across all universes m in $\mathcal{M}$?

It should be obvious that these same questions would also have to be addressed with regard to the more fundamental space of all cosmological wave functions we briefly described earlier, which would probably underlie any ensembles of universes or universe domains drawn from $\mathcal{M}$. It is clear, as we have discussed in EKS, that these questions cannot be answered scientifically, though scientific input is necessary for doing so. How can we answer them philosophically?

2.2 Adequately Specifying Possible Anthropic Universes

When defining any ensemble of universes, possible or realised, we must specify all the parameters which differentiate members of the ensemble from one another at any time in their evolution. The values of these parameters may not be known or determinable initially in many cases – some of them may only be set by transitions that occur via processes like symmetry breaking within given members of the ensemble. In particular, some of the parameters whose values are important for the origination and support of life may only be fixed later in the evolution of universes in the ensemble.

We can separate our set of parameters $\mathcal{P}$ for the space of all possible universes $\mathcal{M}$ into different categories, beginning with the most basic or fundamental, and progressing to more contingent and more complex categories (see EKS). Ideally they should all be independent of one another, but we will not be able to establish that independence for each parameter, except for the most fundamental cosmological ones. In order to categorise our parameters, we can doubly index each parameter p in $\mathcal{P}$ as $p_j(i)$ such that those for $j = 1 - 2$ describe basic physics, for $j = 3 - 5$ describe the cosmology (given that basic physics), and $j = 6 - 7$ pertain specifically to emergence of complexity and of life (see EKS for further details).

Though we did not do so in our first paper EKS, it may be helpful to add a separate category of parameters $p_8(i)$, which would relate directly to the emergence of consciousness and self-conscious life, as well as to the causal effectiveness of self-conscious (human) life – of ideas, intentions and goals. It may turn out that all such parameters may be able to be reduced to those of $p_7(i)$, just as those of $p_6(i)$ and $p_7(i)$ may be reducible to those of physics. But we also may discover, instead, that such reducibility is not possible.

All these parameters will describe the set of possibilities we are able to characterise on the basis of our accumulated scientific experience. This is by no means a statement that “all that can occur” is arbitrary. On the contrary, specifying the set of possible parameters determines a uniform high-level structure that is obeyed by all universes in $\mathcal{M}$.

In the companion cosmology/physics paper to this one (EKS), we develop in detail the geometry, parameters $p_5(i)$, and the physics, parameters $p_1(i)$ to $p_4(i)$, of possible universes. There we also examine in detail the FLRW sector $\mathcal{M}_{FLRW}$ of the ensemble of all possible universes $\mathcal{M}$ to illustrate the relevant mathematical and physical issues. We shall not repeat those discussions here, as they do not directly impact our treatment of the philosophical issues upon which we are focusing. However, since one of the primary motivations for developing the multiverse scenario is to provide a scientific solution to the anthropic fine-tuning problem, we need to discuss briefly the set of ”anthropic” universes.

2.3 The Anthropic subset

The subset of universes that allow intelligent life to emerge is of particular interest. That means we need a function on the set of possible universes that describes the probability that life may evolve. An adaptation of the Drake equation (Drake and Shostak 1998) gives for the expected number of planets with intelligent life in any particular universe m in an ensemble (EKS),

$$N_{\text{life}}(m) = N_g * N_S * \Pi * F, \tag{1}$$

where N_g is the number of galaxies in the model and N_S the average number of stars per galaxy. The probability that a star provides a habitat for life is expressed by the product

$$\Pi = f_S * f_p * n_e \tag{2}$$

and the probability of the emergence of intelligent life, given such a habitat, is expressed by the product

$$F = f_l * f_i. \tag{3}$$

Here f_S is the fraction of stars that can provide a suitable environment for life (they are ‘Sun-like’), f_p is the fraction of such stars that are surrounded by planetary systems, n_e is the mean number of planets in each such system that are suitable habitats for life (they are ‘Earth-like’), f_l is the fraction of such planets on which life actually originates, and f_i represents the fraction of those planets on which there is life where intelligent beings develop. The anthropic subset of a possibility space is that set of universes for which $N_{\text{life}}(m) > 0$.

The quantities $\{N_g, N_S, f_S, f_p, n_e, f_l, f_i\}$ are functions of the physical and cosmological parameters characterised above. So there will be many different representations of this parameter set depending on the degree to which we try to represent such interrelations.

In EKS, following upon our detailed treatment of $\mathcal{M}_{FLRW}$ we identify those FLRW universes in which the emergence and sustenance of life is possible on a broad level[5] – the necessary cosmological conditions have been fulfilled allowing existence of galaxies, stars, and planets if the universe is perturbed, so allowing a non-zero factor $N_g * N_S * \Pi$ as discussed above. The fraction of these that will actually be life-bearing depends on the fulfilment of a large number of other conditions represented by the factor $F = f_l * f_i,$ which will also vary across a generic ensemble, and the above assumes this factor is non-zero.

3 The Set of Realised Universes

We have now characterised the set of possible universes. But in any given existing ensemble, many will not be realised, and some may be realised many times. The purpose of this section is to review our formalism (EKS) for specifying which of the *possible* universes (characterised above) occur in a particular *realised* ensemble.

3.1 A distribution function for realised universes

In order to select from $\mathcal{M}$ a set of realised universes we need to define on $\mathcal{M}$ a distribution function $f(m)$ specifying how many times each type of possible universe m in $\mathcal{M}$ is realised[6]. The function $f(m)$ expresses the contingency in any actualisation – the fact that not every possible universe has to be realised. Things could have been different! Thus, $f(m)$ describes the *ensemble of universes* or *multiverse* envisaged as being realised out of the set of possibilities. In general, these realisations include only a subset of possible universes, and multiple realisation of some of them. Even at this early stage of our discussion we can see that the really existing ensemble of universes is by no means unique.

From a quantum cosmology perspective we can consider $f(m)$ as given by an underlying solution of the Wheeler-de Witt equation, by a given superstring model, or by some other generating mechanism, giving an entangled ensemble of universes or universe domains.

The class of models considered is determined by all the parameters which are held constant ('class parameters'). Considering the varying parameters for a given class ('member parameters'), some will take only discrete values, but for each one allowed to take continuous values we need a volume element of the possibility space $\mathcal{M}$ characterised by parameter increments $\mathrm{d}p_j(i)$ in all such

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[5] More accurately, perturbations of these models can allow life – the exact FLRW models themselves cannot do so.

[6] It has been suggested to us that in mathematical terms it does not make sense to distinguish identical copies of the same object: they should be identified with each other because they are essentially the same. But we are here dealing with physics rather than mathematics, and with real existence rather than possible existence, and then multiple copies must be allowed (for example all electrons are identical to each other; physics would be very different if there were only one electron in existence).

varying parameters $p_j(i)$. The volume element will be given by a product

$$\pi = \Pi_{i,j}\, m_{ij}(m)\, \mathrm{d}p_j(i) \tag{4}$$

where the product $\Pi_{i,j}$ runs over all continuously varying member parameters i, j in the possibility space, and the m_{ij} weight the contributions of the different parameter increments relative to each other. These weights depend on the parameters $p_j(i)$ characterising the universe m. The number of universes corresponding to the set of parameter increments $\mathrm{d}p_j(i)$ will be $\mathrm{d}N$ given by

$$\mathrm{d}N = f(m)\pi \tag{5}$$

for continuous parameters; for discrete parameters, we add in the contribution from all allowed parameter values. The total number of universes in the ensemble will be given by

$$N = \int f(m)\pi \tag{6}$$

(which will often diverge), where the integral ranges over all allowed values of the member parameters and we take it to include all relevant discrete summations. The probable value of any specific quantity $p(m)$ defined on the set of universes will be given by

$$P = \frac{\int p(m) f(m)\pi}{\int f(m)\pi} \tag{7}$$

Such integrals over the space of possibilities give numbers, averages, and probabilities.

Now it is conceivable that all possibilities are realised – that all universes in $\mathcal{M}$ exist at least once. This would mean that the distribution function

$$f(m) \neq 0 \text{ for all } m \in \mathcal{M}.$$

But there are an infinite number of distribution functions which would fulfil this condition. So not even a really existing 'ensemble of all possible universes' is unique. In such ensembles, all possible values of each distinguishing parameter would be represented by its members in all possible combinations with all other parameters at least once. One of the problems is that this means that the integrals associated with such distribution functions would often diverge, preventing the calculation of probabilities.

From these considerations we have the second key issue:

Issue 2: What determines $f(m)$? What is the meta-cause that delimits the set of realisations out of the set of possibilities?

The answer to this question has to be different from the answer to *Issue 1*, precisely because here we are describing the contingency of selection of a subset of possibilities for realisation from the set of all possibilities – determination of the latter being what is considered in *Issue 1.* As we saw in EKS, and as we shall further discuss here (see Section 6), these questions can, in principle, be partially answered scientifically. A really existing ensemble of universes or universe domains demands the operation of a generating process, which adequately explains the origin of its members with their ranges of characteristics and their distribution over the parameters describing them, from a more fundamental potential, a specific primordial quantum configuration, or the decoherence of a specific cosmological wave function. That is, there must be a specific generating process, whatever it is, which determines $f(m)$. When it comes to the further question, what is responsible for the operation of this or that specific generating process rather than some other one which would generate a different ensemble, we see (EKS) that an adequate answer cannot be given scientifically. This is the question why the primordial dynamics leading to the given really existing ensemble of universes is of a certain type rather than of some other type. Even if we could establish $f(m)$ in detail, it is difficult to imagine how we would *scientifically* explain why one generating process was instantiated rather than some other one. The only possibility for an answer, if any, is via philosophical, or possibly theological, considerations.

3.2 Measures and Probabilities

From what we have seen above, it is clear that $f(m)$ will enable us to derive numbers and probabilities relative to the realisation it defines only if we also have determined a unique measure π on the ensemble, characterised by a specific choice of the weights $m_{ij}(m)$ in (4), where these weights will depend on the $p_j(i)$. There are a number of difficult challenges we face in doing this, including the lack of a "natural measure" on $\mathcal{M}$ in all its coordinates, the determination of $f(m)$, or its equivalent, from compelling physical considerations, and the possible divergence of the probability integrals (see Kirchner and Ellis, 2003). These issues have been discussed in EKS.

3.3 The Anthropic subset

The expression (1) can be used in conjunction with the distribution function $f(m)$ of universes to determine the expected number of planets bearing intelligent life arising in the whole ensemble:

$$N_{\text{life}}(E) = \int f(m) * N_g * N_S * f_S * f_p * n_e * f_l * f_i * \pi \tag{8}$$

(which is a particular case of (7) based on (1)). An anthropic ensemble is one for which $N_{\text{life}}(E) > 0$. If the distribution function derives from a probability function, we may combine the probability functions to get an overall anthropic probability function - for an example see Weinberg (2000), where it is assumed that the probability for galaxy formation is the only relevant parameter for the existence of life. This is equivalent to assuming that $N_S * f_S * f_p * n_e * f_l * f_i > 0$.

This assumption might be acceptable in our physically realised Universe, but there is no reason to believe it would hold generally in an ensemble because these parameters will depend on other ensemble parameters, which will vary.

4 Anthropic Parameters, Complexity and Life

The astrophysical issues expressed in the product Π (the lower-j parameters: $j \leq 6$) are the easier ones to investigate anthropically. We can in principle make a cut between those consistent with the eventual emergence of life and those incompatible with it by considering each of the factors in N_g, N_S, and Π in turn, taking into account their dependence on the parameters $p_1(i)$ to $p_5(i)$, and only considering the next factor if all the previous ones are non-zero. In this way we can assign "bio-friendly intervals" to the possibility space $\mathcal{M}$. If $N_g * N_S * \Pi$ is non-zero we can move on to considering similarly whether F is non-zero, based on the parameters $p_6(i)$ to $p_8(i)$, determining if true complexity is possible, which in turn depends on the physics parameters $p_1(i)$ in a crucial way that is not fully understood.

As we go to higher-level parameters we will narrow the number of the number of universes consistent with self-conscious life even more. Essentially, we shall have the sequence of inequalities:

$$N_8 < N_7 < N_6 < N_5 < N_4 < N_3 < N_2 < N_1,$$

where N_j is the total number of universes specified by parameters of level j which are compatible with the eventual emergence of self-conscious life.

This clearly fits very nicely with the Bayesian Inference approach to probability and provides the beginnings of an implementation of it for these multiverses. This approach also clearly keeps the distinction between necessary and sufficient conditions intact throughout. At each level we add to the necessary conditions for complexity or life, weeding out those universes which fail to meet any single necessary condition. Sufficiency is never really reached in our description – we really do not know the full set of conditions which achieve sufficiency. Life demands unique combinations of many different parameter values that must be realised simultaneously. Higher-order ($j \geq 6$) parameters $p_j(i)$ may not even be relevant for many universes or universe domains in a given ensemble, since the structures and processes to which they refer may only be able to emerge for certain very narrow ranges of the lower-j parameters. It may also turn out, as we have already mentioned, that higher-level parameters may be reducible to the lower-level parameters.

It will be impossible at any stage to characterise that set of $\mathcal{M}$ in which *all* the conditions *necessary* for the emergence of self-conscious life and its maintenance have been met, for we do not know what those conditions are (for example, we do not know if there are forms of life possible that are not based on carbon and organic chemistry). Nevertheless it is clear that life demands unique combinations of many different parameter values that must be realised simultaneously, but do not necessarily involve all parameters (for example Hogan [22] suggests that only 8 of the parameters of the standard particle physics model are involved in the emergence of complexity). When we look at these combinations, they will span a very small subset of the whole parameter space (Davies 2003, Tegmark 2003).

5 Problems With Infinity

When speaking of multiverses or ensembles of universes – possible or realised – the issue of infinity inevitably crops up. Researchers often envision an *infinite* set of universes, in which all possibilities are realised. Can there be an infinite set of really existing universes? We suggest that the answer may very well be "No". The common perception that this is possible arises from not appreciating the precisions in meaning and the restrictions in application associated with this profoundly difficult concept. Because we can assign a symbol to represent 'infinity' and can manipulate that symbol according to specified rules, we assume corresponding "infinite" entities can exist in practice. This is questionable[7]. Furthermore, as we have already indicated, such infinities lead to severe calculational problems in the mathematical modelling of ensembles of universes or universe domains, blocking any meaningful application of probability calculus.

It is very helpful to recognize at the outset that there are two different concepts of "the infinite" which are often used: The *metaphysical* infinite, which designates wholeness, perfection, self-sufficiency; and the *mathematical* infinite, which represents that which is without limit (Moore 1990, pp.1-2, 34-44; Bracken 1995, p.142, n.12). Here we are concerned with the mathematical infinite[8]. But, now there are really two basic categories of the mathematical infinite: The potential or conceptual infinite and the actual, or realised, infinite. This distinction goes back to rather diffuse but very relevant discussions by Aristotle in his *Physics* and his *Metaphysics*. Basically, the potential or conceptual infinite refers to a process or set conceptually defined so that it has no limit to it – it goes on and on, e.g. the integers. The concept defining the set or process is without a bound or limit, and open, i. e. it does not repeat or retrace what is already produced or counted. The actual, or realized, infinite would be a concrete real object or entity, or set of objects, which is open and has no limit to its specifications (in space, time, number of components, etc.), no definite upper bound. Aristotle and many others since have argued that, though there are many examples of potential or conceptual infinities, actual realised infinities are not possible as applied to entities or groups of entities.[9]

There is no conceptual problem with an infinite set – countable or uncountable – of *possible* or *conceivable* universes. However, as David Hilbert (1964)

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[7] Our discussion here follows EKS, with the addition of supporting philosophical material and references.

[8] For a fascinating and very readable, but somewhat eccentric, recent history of mathematical infinity and its connections with key mathematical developments, see David Foster Wallace (2003)

[9] Bracken (1995, pp.11-24) gives a recent critical summary of Aristotle's treatment of these issues, and their later use by Thomas Aquinas, Schelling and Heidegger (Bracken, pp.25-51).

points out, the presumed existence of the actually infinite directly or indirectly leads to well-recognised unresolvable contradictions in set theory (e. g., the Russell paradox, involving the set of all sets which do not contain themselves, which by definition must both be a member of itself and not a member of itself!), and thus in the definitions and deductive foundations of mathematics itself (Hilbert, pp.141-142).

Hilbert's basic position is that "Just as operations with the infinitely small were replaced by operations with the finite which yielded exactly the same results . . ., so in general must deductive methods based on the infinite be replaced by finite procedures which yield exactly the same results." (p.135) He strongly maintains that "the infinite is nowhere to be found in reality, no matter what experiences, observations, and knowledge are appealed to." (p.142, see also pp.136-137) Further on he remarks, "Material logical deduction is indispensable. It deceives us only when we form arbitrary abstract definitions, especially those which involve infinitely many objects. In such cases we have illegitimately used material logical deduction; i.e., we have not paid sufficient attention to the preconditions necessary for its valid use." (p.142). Hilbert concludes, "Our principal result is that the infinite is nowhere to be found in reality. It neither exists in nature nor provides a legitimate basis for rational thought . . . The role that remains for the infinite to play is solely that of an idea . . . which transcends all experience and which completes the concrete as a totality . . ." (Hilbert, p.151).

5.1 Arguments against Actual Infinity

What are we to make of these intuitions and arguments? There are many mathematicians and philosophers who espouse them. There are also a large number who maintain that they are flawed. From the point of view of cosmology itself, it would be very helpful if we could trust in the conclusion that an actualized mathematical infinity is physically impossible. For then this would provide a constraint on the scenarios we use in cosmology, and assure us that probability calculations using them could be successfully pursued. If, instead, there emerges a clear indication that actual infinite sets are possible, that would be mathematically disappointing. However, it still would be an important conclusion, providing guidance and reassurance in our quest to understand not only our observable universe, but the universe or multiverse as whole, even though we will never have *direct* access to all of it (see Section 7, below).

It is not possible to explore this issue conclusively here. Philosophers and philosophers of mathematics and science have proposed many arguments against the possibility of realized or actual mathematical infinities, and many others, arguments in their favor. A careful critical review is far beyond the scope of this paper. We need, however, to go beyond the general and somewhat unfocused reasonings of Aristotle, Hilbert and others we have summarized in the introduction to this section. Thus, we shall briefly but more carefully present several arguments against actual mathematical infinities which we consider the strongest. Then in the next section we shall explore in detail some of the mathematical and physical reasons for which we should avoid admitting actual infinities.

We begin by proposing several key definitions. An *actually existing set* is one which has concrete physical status in our extra-mental, intersubjective experience, and each of whose members has a determinate, phenomenally supported status in our experience, distinct from other members, with physically characterisable features and integrity (e. g., a certain mass or energy, etc.). If the members of the set are not distinct or determinate, then the set is either not well-defined or is not actually existing.[10] Any such actually existing set, and the members constituting it, is contingent. They depend on something else for being the way they are. It came into existence at a certain moment, or within a certain series of moments, as the result of a certain process, and eventually dissipates or dissolves, gradually changing into something else over time. This is a pervasive feature of our experience and of our scientific investigation of physical reality.

An *infinite set* is, as we have already said, one the number of whose members is open, indeterminate and unbounded. By "indeterminate" we mean unspecified in terms of a definite number. Infinity, strictly speaking is not a number in any usual sense – it is beyond all specific numbers which might be assigned to a set or system – it is simply the code-word for "it continues without end". This definition reflects how the term "infinity" is used in mathematical physics, and in most of mathematics. The key point is that there is no specific number which can be assigned to the number of elements in an infinite set. One could say that the number of its elements is one of Cantor's transfinite numbers, but those are not numbers which specify a determinate bound.

An *infinite physically existing set* is an infinite set which is also an actually existing set – in other words it possesses an infinity of really existing objects. The number of its objects is therefore unbounded and indeterminate. That means in essence that, though it is unbounded and indeterminate in number, it is nevertheless physically realized as complete. One should note at this point that the definition is contradictory, not from a logical or mathematical point of view (see Stoeger 2004), but from the point of view of physics and metaphysics. Can what is essentially indeterminate and unbounded be physically or really complete, which seems to imply "bounded"? Can anything which physically exists be completely unbounded? It is clear from our definition of infinity that it is not a specific number we can determine. Not only is it beyond any number we can specify or conceive. It is unboundedly large and indeterminate. But an actual infinity is conceived as extra-mentally instantiated and therefore as completed. That means it must be determinate and in some definite sense bounded. But this contradicts the definition of what infinity is. Something cannot be bounded and determinate, and unbounded and indeterminately large, at the same time. Therefore, an actual physically realized infinity is not possible.

This appears, on the face of it, to be a compelling conceptual argument

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[10] In quantum theory, the members of a set (e. g. particles) may be virtual, going into and out of existence, but at any one time there are only "so many." Furthermore, there is a number operator, even though the particles themselves cannot be physically distinguished.

against an actual really existing infinite set of objects, whether they be universes, or something else. The only way to counter it would be to show that an indeterminate unboundedly large set is physically realisable. Conceptually this seems to be impossible, just from the point of view of what we mean by physically concrete or actual, which seems to demand specifications and some boundedness. Something which is unboundedly large, and therefore not specifiable or determinate in quantity or extent, is not materially or physically realisable. It is not just that we are incapable of knowing an actual infinity. It seems to involve a definite physical impossibility – the unbounded indeterminateness essential to infinity is inconsistent with what it means to be physically instantiated.

Another way of putting this is that the definition of infinity is an issue in mathematics, not in physics. The problem arises in linking the well-defined mathematical concept of infinity with attempts at its realisation in physics. Realisations in physics must have some determinateness – but infinity as such is not determinate!

This is recognized implicitly in scientific and applied mathematics practice. Whenever infinite values of physical parameters arise in physics – such as in the case of singularities – we can be reasonably sure, as is often indicated, that there has been a breakdown in our models. An achieved infinity in any physical parameter (temperature, density, spatial curvature) is almost certainly *not* a possible outcome of any physical process – simply because it is unboundly large, indefinite and indeterminate.

A second supporting argument against realized infinity can be constructed as follows. Since a realised infinite set of objects is actually existing as a physical set, it is contingent and therefore must have come into existence by some generating process.[11] Then there are two possibilities: 1. it became an actual infinite set by some process of successive addition; or 2. it was produced as an infinite set all at once.

But 1. does not work, since one cannot achieve a physically infinite set by successive addition – we can never actually *arrive* at infinity that way (see Spitzer 2000 and Stoeger 2004, and references therein). There is no physical process or procedure we can in principle implement to complete such a set – they are simply incompletable. Some will concede that we can never physically arrive at infinity in a finite time (see Smith 1993), but maintain we can do so in an infinite time. But then we have the same problem again with time – that, for that to happen, we must *complete* an infinite number of events. But that seems to contradict the essentially unbounded and indeterminate character of "infinite time."

So that leaves us with possibility 2., that the infinite set was physically produced all at once. This is the one possibility Bertrand Russell admits (Russell 1960). But, to produce an infinite realized set of physical objects all at once requires a process which makes an actually infinite amount of mass-energy

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[11] When contemplating mathematical concepts, it is debatable as to whether a procedure or process is needed. But we are talking physics, and the issue is precisely whether or not the concept is realisable in the physical sense.

available. Again this must be a real complete, specified, infinite amount of mass-energy. But this seems conceptually contradictory again, for similar reasons.

Furthermore, the specification “all at once” demands simultaneity, which is totally coordinate dependent. What is simultaneous with respect to one coordinate system is not simultaneous with respect to another. Thus, there is no assurance to begin with that one can avoid the temporal completion problem with 1. above; indeed one cannot do so with respect to all coordinate systems. Therefore, once again here on several counts it seems that a really infinite set of physical objects is not realisable or actualizable.

These arguments underscore the fact that the problem with a realised infinity is not primarily physical in the usual sense – it is primarily a conceptual or philosophical problem. “Infinity” as it is mathematically conceived and used, is not the sort of property that can be physically realised in an entity, an object, or a system, like a definite number can. It is indeterminately large, and really refers to a process rather than to an entity (Bracken 1995, pp. 11-24). And the process it refers to has no term or completion specified. No physically meaningful parameter really possesses an infinite value. It is true that cosmologists and physicists use infinities in ways which seem to border on realised infinities, such as an infinite number of points in a line segment, an infinite number of directions from any point in three-space, or an infinite dimensional Hilbert space.[12] However, these are potential infinities, indicating possible directions, locations or states that could be taken or occupied. In no case are they all realised, occupied or taken by distinguishable, really existing entities.

Finally, it is worth emphasizing that actual physically realized infinities lead to a variety of apparently irresolvably paradoxical, if not contradictory results (see Craig 1993) in thought experiments, such as those involving adding to and borrowing books from a really infinite library, or putting up new guests in an already fully occupied hotel of an infinite number of rooms. In fact, just the notion of a completed infinite set seems to underlie some of the disturbing paradoxes of set theory (see Craig 1993 for a brief discussion and references).

This issue is distinct from the difference between an ontologically realised infinity or an epistemologically realised infinity. What we have presented above seems to undermine the possibility of the former, at least as a physical possibility. But it is a separate question whether or not, granted the existence of a physically realized infinity, it could ever be known or specified as such in a completed and determinate way.

5.2 Actual Infinities in Cosmology?

Whether or not actual infinities are possible, they certainly need to be avoided on the physical level, in order to make progress in studying multiverses. As we have already discussed, actual infinities lead to irresolvable problems in making probability calculations; and their existence or non-existence is certainly not observationally provable. They are an untestable proposal.

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[12] We thank an anonymous referee for pointing out these examples.

In the physical universe spatial infinities can be avoided by compact spatial sections, either resultant from positive spatial curvature or from choice of compact topologies in universes that have zero or negative spatial curvature, (for example FLRW flat and open universes can have finite rather than infinite spatial sections). We argue that the theoretically possible infinite space sections of many cosmologies at a given time are simply unattainable in practice - they are a theoretical idea that cannot be realised. It is certainly unprovable that they exist, if they do. However one can potentially get evidence against such infinities - if either it is observationally proven that we live in a a 'small universe', where we have already seen round the universe because the spatial sections are compact on a scale smaller than the Hubble scale (Lachieze-Ray and Luminet 1995);[13] or if we prove that the spatial curvature of the best-fit FLRW universe model is positive, which necessarily implies closed spatial sections (see Sec.7.4 below).

Future infinite time also is never realised: rather the situation is that whatever time we reach, there is always more time available.[14] Much the same applies to claims of a past infinity of time: there may be unbounded time available in the past in principle, but in what sense can it be attained in practice? The arguments against an infinite past time are strong - it is simply not constructible in terms of events or instants of time, besides being conceptually indefinite.[15]

The same problem of a realised infinity appears in considering supposed ensembles of really existing universes. Aside from the strictly philosophical issues we have discussed above, conceiving of an ensemble of many 'really existing' universes that are totally causally disjoint from our own, and how that could come into being presents a severe challenge to cosmologists. There are two fundamental reasons for this. First, specifying the geometry of a generic universe requires an infinite amount of information because the quantities in $\mathcal{P}_{\text{geometry}}$ are fields on spacetime, in general requiring specification at each point (or equivalently, an infinite number of Fourier coefficients) - they will almost always not be algorithmically compressible. This greatly aggravates all the problems regarding infinity and the ensemble itself. Only in highly symmetric cases, like the FLRW solutions, does this data reduce to a finite number of parameters. One can suggest that a statistical description would suffice, where a finite set of numbers describes the statistics of the solution, rather than giving a full description. Whether this suffices to describe adequately an ensemble where 'all

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[13] There are some observational indications that this could be so (see Sec.(7.4)), but they are far from definitive.

[14] Obviously this does not mean that we reject standard Big Bang cosmology – rejecting the really spatially infinite universes as unrealizable does not undermine the observational adequacy of these models, nor the essence of the Big Bang scenario, even in the cases of those which are flat or open. It just indicates that these models are incomplete, which we already recognized.

[15] One way out would be, as quite a bit of work in quantum cosmology seems to indicate, to have time originating or emerging from the quantum-gravity dominated primordial substrate only "later." In other words, there would have been a "time" or an epoch before time as such emerged. Past time would then be finite, as seems to be demanded by philosophical arguments, and yet the timeless primordial state could have lasted "forever," whatever that would mean. This possibility avoids the problem of constructibility.

that can happen, happens' is a moot point. We suggest not, for the simple reason that there is no guarantee that all possible models will be included in any known statistical description. That assumption is a major restriction on what is assumed to be possible.

Secondly, if many universes in the ensemble themselves really have infinite spatial extent and contain an infinite amount of matter, that entails certain deeply paradoxical conclusions (Ellis and Brundrit 1979). To conceive of the physical creation of an infinite set of universes (most requiring an infinite amount of information for their prescription, and many of which will themselves be spatially infinite) is at least an order of magnitude more difficult than specifying an existent infinitude of finitely specifiable objects.

The phrase 'everything that can exist, exists' implies such an infinitude, but glosses over all the profound difficulties implied. One should note here particularly that problems arise in this context in terms of the continuum assigned by classical theories to physical quantities and indeed to spacetime itself. Suppose for example that we identify corresponding times in the models in an ensemble and then assume that *all* values of the density parameter occur at each spatial point at that time. On the one hand, because of the real number continuum, this is an uncountably infinite set of models – and, as we have already seen, genuine existence of such an uncountable infinitude is highly problematic. But on the other hand, if the set of realised models is either finite or countably infinite, then almost all possible models are not realised – the ensemble represents a set of measure zero in the set of possible universes. Either way the situation is distinctly uncomfortable.

However, we might try to argue around this by a discretization argument: maybe differences in some parameter of less than say 10^{-10} are unobservable, so we can replace the continuum version by a discretised one, and perhaps some such discretisation is forced on us by quantum theory - indeed this is a conclusion that follows from loop quantum gravity, and is assumed by many to be the case whether loop quantum gravity is the best theory of quantum gravity or not. That solves the 'ultraviolet divergence' associated with the small-scale continuum, but not the 'infrared divergence' associated with supposed infinite distances, infinite times, and infinite values of parameters describing cosmologies.

Even within the restricted set of FLRW models, the problem of realised infinities is profoundly troubling: if all that is possible in this restricted subset happens, we have multiple infinities of realised universes in the ensemble. First, there are an infinite number of possible spatial topologies in the negative curvature case (see e.g. Lachieze-Ray and Luminet 1995), so an infinite number of ways that universes which are locally equivalent can differ globally. Second, even though the geometry is so simple, the uncountable continuum of numbers plays a devastating role locally: is it really conceivable that FLRW universes actually occur with *all* values independently of both the cosmological constant and the gravitational constant, and also all values of the Hubble constant, at the instant when the density parameter takes the value 0.97? This gives 3 separate uncountably infinite aspects of the ensemble of universes that are supposed to exist. Again, the problem would be allayed if spacetime is quantized at the Planck level, as suggested for example by loop quantum gravity. In that case one can argue that all physical quantities also are quantized, and the uncountable infinities of the real line get transmuted into finite numbers in any finite interval – a much better situation. We believe that this is a physically reasonable assumption to make, thus softening a major problem for many ensemble proposals. But the intervals are still infinite for many parameters in the possibility space. Reducing the uncountably infinite to countably infinite does not in the end resolve the problem of infinities in these ensembles. It is still an extraordinarily extravagant proposal, and, as we have just discussed, seems to founder in the face of careful conceptual analysis.

The argument given so far is based in the nature of the application of mathematics to the description of physical reality. We believe that it carries considerable weight, even though the ultimate nature of the mathematics-physics connection is one of the great philosophical puzzles. It is important to recognize, however, that arguments regarding problems with realised infinity arise from the physics side, independently of the mathematical and conceptual consideration we have so far emphasized.

On one hand, broad quantum theoretical considerations suggest that spacetime may be discrete at the Planck scale, and some specific quantum gravity models indeed have been shown to incorporate this feature when examined in detail. If this is so, not only does it remove the real number line as a *physics* construct, but it *inter alia* has the potential to remove the ultraviolet divergences that otherwise plague field theory – a major bonus.

On the other hand, it has been known for a long time that there are significant problems with putting boundary conditions for physical theories actually at infinity. It was for this reason that Einstein preferred to consider universe models with compact spatial sections (thus removing the occurrence of spatial infinity in these models). This was a major motivation for his static universe model proposed in 1917, which necessarily has compact space sections. John Wheeler picked up this theme, and wrote about it extensively in his book *Einstein's Vision* (1968). Subsequently, the book *Gravitation* by Misner, Thorne and Wheeler (1973) only considered spatially compact, positively curved universe models in the main text. Those with flat and negative spatial curvatures where relegated to a subsection on "Other Models."

Thus, this concern regarding infinity has a substantial physics provenance, independent of Hilbert's mathematical arguments and philosophical considerations. It recurs in present day speculations on higher dimensional theories, where the higher dimensions are in many cases assumed to be compact, as in the original Kaluza-Klein theories. Various researchers have then commented that "dimensional democracy" suggests all spatial sections should be compact, unless one has some good physical reason why those dimensions that remained small are compact while those that have expanded to large sizes are not. Hence we believe there is substantial support from physics itself for the idea that the universe may have compact spatial sections, thus also avoiding infra-red divergences – even though this may result in "non-standard" topologies for its spatial sections. Such topologies are commonplace in string theory and in M-theory – indeed they are essential to their nature.

6 On the origin of ensembles

Ensembles have been envisaged both as resulting from a single causal process, and as simply consisting of discrete entities. We discuss these two cases in turn, and then show that they are ultimately not distinguishable from each other.

6.1 Processes Naturally Producing Ensembles

Over the past 15 or 20 years, many researchers investigating the very early universe have proposed processes at or near the Planck era which would generate a really existing ensemble of expanding universe domains, one of which is our own observable universe. In fact, their work has provided both the context and stimulus for our discussions in this paper. Each of these processes essentially selects a really existing ensemble from a set of possible universes, often leading to a proposal for a natural definition of a probability distribution on the space of possible universes. Here we briefly describe some of these, and comment on how they fit within the framework we have been discussing.

The earliest explicit proposal for an ensemble of universes or universe domains was by Vilenkin (1983). Andrei Linde's chaotic inflationary proposal (Linde 1983, 1990, 2003) is one of the best known scenarios of this type. The scalar field (inflaton) in these scenarios drives inflation and leads to the generation of a large number of causally disconnected regions of the Universe. This process is capable of generating a really existing ensemble of expanding FLRW-like regions, one of which may be our own observable universe region, situated in a much larger universe that is inhomogeneous on the largest scales. No FLRW approximation is possible globally; rather there are many FLRW-like sub-domains of a single fractal universe. These domains can be very different from one another, and can be modelled locally by FLRW cosmologies with different parameters.

Vilenkin, Linde and others have applied a stochastic approach to inflation (Vilenkin 1983, Starobinsky 1986, Linde, *et al.* 1994, Vilenkin 1995, Garriga and Vilenkin 2001, Linde 2003), through which probability distributions can be derived from inflaton potentials along with the usual cosmological equations (the Friedmann equation and the Klein-Gordon equation for the inflaton) and the slow-roll approximation for the inflationary era. A detailed example of this approach, in which specific probability distributions are derived from a Langevin-type equation describing the stochastic behaviour of the inflaton over horizon-sized regions before inflation begins, is given in Linde and Mezhlumian (2003) and in Linde *et al.* (1994). The probability distributions determined in this way generally are functions of the inflaton potential.

As we mentioned in the introduction, over the past few years considerable progress has been achieved by theorists in developing flux stabilized, compactified, non-supersymmetric solutions to superstring/M theory which possess an enormous number of vacua (Susskind 2003, Kachru, et al. 2003, and references therein). Each of these vacua has the potential for becoming a separate universe or universe domain, with a non-zero cosmological constant. As such it is relatively easy to initiate inflation in many of them. Furthermore, the dynamics leading to these vacua also generate different values of the some of the other cosmological and physical parameters, and enable a statistical treatment of string vacua themselves.

These kinds of scenario suggests how overarching physics, or a "law of laws"(represented by the inflaton field and its potential), can lead to a really existing ensemble of many very different FLRW-like regions of a larger Universe. However these proposals rely on extrapolations of presently known physics to realms far beyond where its reliability is assured. They also employ inflaton potentials which as yet have no connection to the particle physics we know at lower energies. And these proposals are not directly observationally testable – we have no astronomical evidence that the supposed other FLRW-like regions exist, and indeed do not expect to ever attain such evidence. Thus they remain theoretically based proposals rather than provisionally acceptable models – much less established fact. There remains additionally the difficult problem of infinities, which we have just discussed: eternal inflation with its continual reproduction of different inflating domains of the Universe is claimed to lead to an infinite number of universes of each particular type (Linde, private communication). How can one deal with these infinities in terms of distribution functions and an adequate measure? As we have pointed out above, there is a philosophical problem surrounding a realised infinite set of any kind. In this case the infinities of really existent FLRW-like domains derive from the assumed initial infinite flat (or open) space sections - and we have already pointed out the problems in assuming such space sections are actually realised. If this is correct, then at the very least these proposals must be modified so that they generate a finite number of universes or universe domains.

Finally, from the point of view of the ensemble of all possible universes often invoked in discussions of multiverses, all possible inflaton potentials should be considered, as well as all solutions to all those potentials. They should all be represented in $\mathcal{M}$, which will include chaotic inflationary models which are stationary as well as those which are non-stationary. Many of these potentials may yield ensembles which are uninteresting as far as the emergence of life is concerned, but some will be bio-friendly.

In EKS we have briefly reviewed various proposals for probability distributions of the cosmological constant over ensembles of universe domains generated by the same inflaton potential, particularly those of Weinberg (2000) and Garriga and Vilenkin (2000, 2001). We shall not revisit this work here, except to mention the strong anthropic constraints on values of the cosmological constant, which is the primary reason for interest in this case. Galaxy formation is only possible for a narrow range of values of the cosmological constant, Λ, around $\Lambda = 0$ (one order of magnitude - hugely smaller than the 120 orders of magnitude predicted by quantum field theory as its natural value).

6.2 Testability of these proposals

In his popular book *Our Cosmic Habitat* Martin Rees (Rees 2001b, pp. 175ff) uses this narrow range of bio-friendly values of Λ to propose a preliminary test which he claims could rule out the multiverse explanation of fine-tuning for certain parameters like Λ. This is what might be called a "speciality argument." According to Rees, if "our universe turns out to be *even more specially* tuned than our presence requires," the existence of a multiverse to explain such "overtuning" would be refuted. The argument itself goes this way. Naive quantum physics expects Λ to be very large. But our presence in the universe requires it to be very small, small enough so that galaxies and stars can form. Thus, in our universe Λ must obviously be below that galaxy-forming threshold. This explains the observed very low value of Λ as a selection effect in an existing ensemble of universes. Although the probability of selecting at random a universe with a small Λ is very small, it becomes large when we add the prior that life exists. Now, in any universe in which life exists, we would not expect Λ to be too far below this threshold. Otherwise it would be more fine-tuned than needed. In fact, data presently indicates that Λ is not too far below the threshold, and thus our universe is not markedly more special than it needs to be, as far as Λ is concerned. Consequently, explaining its fine-tuning by assuming a really existing multiverse is acceptable. Rees suggests that the same argument can be applied to other parameters.

Is this argument compelling? As Hartle (2004) has pointed out, for the first stage to be useful, we need an *a priori* distribution for values of Λ that is very broad, combined with a very narrow set of values that allow for life. These values should be centred far from the most probable *a priori* values. This is indeed the case if we suppose a very broad Gaussian distribution for Λ centred at a very large value, as suggested by quantum field theory. Then, regarding the second stage of the argument, the values allowing for life fall within a very narrow band centred at zero, as implied by astrophysics. Because the biophilic range is narrow, the *a priori* probability for Λ will not vary significantly in this range. Thus, it is equally likely to take any value. Thus a *uniform probability assumption* will be reasonably well satisfied within the biophilic range of Λ.

As regards this second stage of the argument, because of the uniform probability assumption it is not clear why the expected values for the existence of galaxies should pile up near the biological limit. Indeed, one might expect the probability of the existence of galaxies to be maximal at the centre of the biophilic range rather than at the edges (this probability drops to zero at the edges, because it vanishes outside – hence the likelihood of existence of galaxies at the threshold itself should be very small). Thus there is no justification on this basis for ruling out a multiverse with any specific value for Λ within that range. As long as the range of values of a parameter like Λ is not a zero-measure set of the ensemble, there is a non-zero probability of choosing a universe within it. In that case, there is no solid justification for ruling out a multiverse and so no real testability of the multiverse proposal. All we can really say is that we would be less likely to find ourselves in a universe with a Λ in that range in that particular ensemble. Indeed no probability argument can conclusively *disprove* any specific result - all it can state is that the result is improbable - but that statement only makes sense if the result is possible! What is actually meant by "more specially tuned than necessary for our existence"? In the end, any particular choice of a life-allowing universe will be more specially tuned for something. In our view "tuning" refers to parameters selected such that the model falls into a certain class, e.g., life-allowing. Any additional tuning would then just be the selection of sub-classes, and, after all, any particular model is "over-tuned" in such a way as to select uniquely the sub-class which contains only itself. Rees's argument seems to imply that Λ close to zero would be an over-tuned case, while Λ close to the cut-off value would not be. However, would the reversed viewpoint be not just as legitimate?

Rees's argument strongly builds on the predictions of quantum physics — a probability distribution peaked at very high values for Λ. Taking into account the unknown relation between general relativity and quantum physics we should treat the problem as a multiple hypothesis testing problem: The multiverse scenario can be true or false, and so can the quantum prediction for high values of Λ. An observed low value of Λ would then strongly question the predictions for Λ, but say nothing about the multiverse scenario. We conclude that any observed value of Λ does not rule out the multiverse scenario. It also seems questionable whether the life-allowing values for Λ can be classified just by a simple cut-off value. It should be expected that there are more subtle and yet unknown constraints. Observing a cosmological constant far from the cut-off value might then just be the result of some unknown constraints.

Finally, probability arguments simply don't apply if there is indeed only one universe - their very use assumes a multiverse exists. There might exist only one universe which just happens to have the observed value of Λ; then probabilistic arguments will simply not apply. Thus what we are being offered here is not in fact a proof a multiverse exists, but rather a consistency check as regards the nature of the proposed multiverse. It is a proposal for a necessary but not sufficient condition for its existence. As emphasized above, we do not even believe it is a necessary condition; rather it is a plausibility indicator.

6.3 The existence of regularities

Consider now a genuine multiverse. Why should there be any regularity at all in the properties of universes in such an ensemble, where the universes are completely disconnected from each other? If there are such regularities and specific resulting properties, this suggests a mechanism creating that family of universes, and hence a causal link to a higher domain which is the seat of processes leading to these regularities. This in turn means that the individual universes making up the ensemble are not actually independent of each other. They are, instead, products of a single process, or meta-process, as in the case of chaotic inflation. A common generating mechanism is clearly a causal connection, even if not situated in a single connected spacetime – and some such mechanism is needed if all the universes in an ensemble have the same class of properties, for example being governed by the same physical laws or meta-laws.

The point then is that, as emphasized when we considered how one can describe ensembles, any multiverse with regular properties that we can characterise systematically is necessarily of this kind. If it did not have regularities of properties across the class of universes included in the ensemble, we could not even describe it, much less calculate any properties or even characterise a distribution function.

Thus in the end the idea of a completely disconnected multiverse with regular properties but without a common causal mechanism of some kind is not viable. There must necessarily be some pre-realisation causal mechanism at work determining the properties of the universes in the ensemble. What are claimed to be totally disjoint universes must in some sense indeed be causally connected together, albeit in some pre-physics or meta-physical domain that is causally effective in determining the common properties of the universes in the multiverse. This is directly related to the two key issues we highlighted above in Sections 2 and 3, respectively, namely how does the possibility space originate, and where does the distribution function that characterises realised models come from?

From these considerations, we see that we definitely need to explain (for Issue 2) what particular cosmogonic generating process or meta-law pre-exists, and how that process or meta-law was selected from those that are possible. Obviously an infinite regress lurks in the wings. Though intermediate scientific answers to these questions can in principle be given, it is clear that no ultimate scientific foundation can be provided.

Furthermore, we honestly have to admit that any proposal for a particular cosmic generating process or principle we establish as underlying our actually existing ensemble of universe domains or universes, after testing and validation (see Section 7 below), will always be at best provisional and imperfect: we will never be able to definitively determine its nature or properties. The actually existing cosmic ensemble may in fact be much, much larger – or much, much smaller – than the one our physics at any given time describes, and embody quite different generating processes and principles than the ones we provisionally settle upon. This is particularly true as we shall never have direct access to the ensemble we propose, or to the underlying process or potential upon which its existence relies (see Section 7 below), nor indeed to the full range of physics that may be involved.

6.4 The existence of possibilities

Turning to the prior question (Issue 1, see Section 2.1), what determines the space of all possible universes, from which a really existing universe or an ensemble of universes or universe domains is drawn, we find ourselves in even much more uncertain waters. This is particularly difficult when we demand some basic meta-principle which delimits the set of possibilities. Where would such a principle originate? The only two secure grounds for determining possibility are existence (”ab esse ad posse valet illatio”) and freedom from internal contradiction. The first really does not help us at all in exploring the boundaries of the possible. The second leaves enormous unexplored, and probably unexplorable, territory. There are almost certainly realms of the possible which we cannot even imagine. But at the same time, there may be, as we have already mentioned, universes we presently think are possible which are not. We really do not have secure grounds for determining the limits of possibility in this expanded cosmic context. We simply do not have enough theoretical knowledge to describe and delimit reliably the realm of the possible, and it is very doubtful we shall ever have.

7 Testability and Existence

The issue of evidence and testing has already been briefly mentioned. This is at the heart of whether an ensemble or multiverse proposal should be regarded as physics or as metaphysics.

7.1 Evidence and existence

Given all the possibilities discussed here, which specific kind of ensemble is claimed to exist? Given a specific such claim, how can one show that this is the particular ensemble that exists rather than all the other possibilities?

There is no direct evidence of existence of the claimed other universe regions in an ensemble, nor can there be any, for they lie beyond the visual horizon; most will even be beyond the particle horizon, so there is no causal connection with them; and in the case of a true multiverse, there is not even the possibility of any indirect causal connection - the universes are then completely disjoint and nothing that happens in any one of them is causally linked to what happens in any other one (see Section 6.2). This lack of any causal connection in such multiverses really places them beyond any scientific support – there can be no direct or indirect evidence for the existence of such systems. We may, of course, postulate the existence of such a multiverse as a metaphysical assumption, but it would be a metaphysical assumption without any further justifiability – it would be untestable and unsupportable by any direct or indeed indirect evidence.

And so, we concentrate on possible really existing multiverses in which there is some common causal generating principle or process. What weight does a claim of such existence carry in this case, when no direct observational evidence can ever be available? The point is that there is not just an issue of showing a multiverse exists. If this is a scientific proposition one needs to be able to show which specific multiverse exists; but there is no observational way to do this. Indeed if you can't show *which particular* one exists, it is doubtful you have shown *any* one exists. What does a claim for such existence mean in this context? Gardner puts it this way: "There is not the slightest shred of reliable evidence that there is any universe other than the one we are in. No multiverse theory has so far provided a prediction that can be tested. As far as we can tell, universes are not even as plentiful as even *two* blackberries" (Gardner 2003).

This contrasts strongly, for example, with Deutsch's and Lewis's defence of the concept (Deutsch 1998, Lewis 2000).

7.2 Fruitful Hypotheses and evidence

There are, however, ways of justifying the existence of an entity, or entities, like a multiverse, even though we have no direct observations of it. Arguably the most compelling framework within which to discuss testability is that of "retroduction" or "abduction" which was first described in detail by C.S.Peirce. Ernan McMullin (1992) has convincingly demonstrated that retroduction is the rational process by which scientific conclusions are most fruitfully reached. On the basis of what researchers know, they construct imaginative hypotheses, which are then used to probe and to describe the phenomena in deeper and more adequate ways than before. As they do so, they will modify or even replace the original hypotheses, in order to make them more fruitful and more precise in what they reveal and explain. The hypotheses themselves may often presume the existence of certain hidden properties or entities (like multiverses!) which are fundamental to the explanatory power they possess. As these hypotheses become more and more fruitful in revealing and explaining the natural phenomena they investigate, and their inter-relationships, and more central to scientific research in a given discipline, they become more and more reliable accounts of the reality they purport to model or describe. Even if some of the hidden properties or entities they postulate are never directly detected or observed, the success of the hypotheses indirectly leads us to affirm that something like them must exist.[16] A cosmological example is the inflaton supposed to underlie inflation.

Thus, from this point of view, the existence of an ensemble of universes or universe domains would be a validly deduced – if still provisional – scientific conclusion if this becomes a key component of hypotheses which are successful and fruitful in the long term. By an hypothesis which manifests long-term success and fruitfulness we mean one that better enables us to make testable predictions which are fulfilled, and provides a more thorough and coherent explanation of phenomena we observe than competing theories.[17] Ernan McMullin (1992; see also P. Allen 2001, p. 113) frames such fruitfulness and success as:

a. accounting for all the relevant data (empirical adequacy);
b. providing long-term explanatory success and stimulating fruitful lines of

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[16] In light of discussions by McMullin elsewhere (McMullin 1993, pp. 381-382) more care and precision is needed here. He recommends separating explanation from proof of existence: "In science, the adequacy of a theoretical explanation is often regarded as an adequate testimony to the existence of entities postulated by the theory. But the debates that swirl around this issue among philosophers (the issue of scientific realism, as philosophers call it) ought to warn us of the risks of moving too easily from explanatory adequacy to truth-claims for the theory itself. This sort of inference depends sensitively on the quality of the explanation given, on the viability of alternatives, on our prior knowledge of beings in the postulated category, and on other more complex factors."

[17] It is interesting to note that Rees (2001b, p. 172) hints at the use of a retroductive approach in cosmology, but does not develop the idea as an argument in any detail.

further inquiry (theory fertility);

c. establishing the compatibility of previously disparate domains of phenomena (unifying power);

d. manifesting consistency and correlation with other established theories (theoretical coherence).

The relevant example here would be a fruitful theory relying on a specific type of multiverse, all members of which would never be directly detectable except one. But, since its postulated existence renders the existence and the characteristic features of our own universe ever more intelligible and coherent over a period of time, this can be claimed to be evidence for the multiverse's existence. If such indirect support for the existence of a given multiverse is inadequate in the light of other competing accounts, then from a scientific point of view all we can do is to treat it as a speculative scenario needing further development and requiring further fruitful application. Without that, espousing the existence of a given multiverse as the explanation for our life-bearing universe must surely be called metaphysics, because belief in its existence will forever be a matter of faith rather than proof or scientific support.

We do, of course, want to avoid sliding to the bottom of Rees' (2001b, p. 169) slippery slope. In arriving at his conclusion that the existence of other universes is a scientific question, Rees (2001b, pp. 165-169) begins by considering first galaxies which are beyond the limits of present-day telescopes, and then galaxies which are beyond our visual horizon now, but will eventually come within it in the future. In both cases these galaxies are real and observable *in principle.* Therefore, they remain legitimate objects of scientific investigation. However, then he goes on to consider galaxies which are forever unobservable, but which emerged from the same Big Bang as ours did. And he concludes that, though unobservable, they are real, and by implication should be included as objects considered by science. Other universes, he argues, fall in the same category – they are real, and therefore they should fall within the boundaries of scientific competency. As articulated this is indeed "a slippery slope" argument – it can be used to place anything that we claim to be "real" within the natural sciences – unless we strengthen it at several points.

First, Rees shifts the criterion from "observable in principle" to being "real." This is really an error. No matter how real an object, process, or relationship may be, if it is not observable in principle, or if there is not at least indirect support for its existence from the long-term success of the hypotheses in which it figures, then it simply falls outside serious scientific consideration. It may still temporarily play a role in scientific speculation, but, unless it receives some evidential support, that will not last. In mentioning that the forever unobservable galaxies he is considering are produced by the same Big Bang as ours, Rees may be intending to indicate that, though unobservable, they share a common causal origin and therefore *figure* in successful hypotheses, as would be required by McMullin's retroductive inference discussed above. But Rees does not make that clear. Moving to other universes, the same requirement holds. Thus, the slippery slope is avoided precisely by implementing the "indirect evidence by fruitful hypotheses" approach that a careful application of retroduction requires.

Second, there is discontinuity in the argument as one moves from weaker and weaker causal relation to none at all. The slippery slope becomes a vertical precipice on one side of an unbridgeable gulf. An argument that relies on incremental continuity does not apply in this case.

Thus, if we are continually evaluating our theories and speculations with regard to their potential and actual fruitfulness in revealing and explaining the world around us, then we shall avoid the lower reaches of the slippery slope. The problem is that, in this case, the multiverse hypothesis is very preliminary and will probably always remain provisional. This should not prevent us from entertaining imaginative scenarios, but the retroductive process will subject these speculations to rigorous critique over time. The key issue then is to what degree will the multiverse hypothesis become fruitful. Unfortunately, as it stands now, it is not, because it can be used to explain anything at all – and hence does not explain anything in particular. You cannot predict something new from the hypothesis, but you can explain anything you already know. In order for it to achieve some measure of scientific fruitfulness, there must be an accumulation of at least indirect scientifically acceptable support for one particular well-defined multiverse. Indeed, from a purely evidential viewpoint, a multiverse with say 10^{120} identical copies of the one universe in which we actually live would be much preferred over one with a vast variety of different universes, for then the probability of finding a universe like our own would be much higher. Such ensembles are usually excluded because of some hidden assumptions about the nature of the generating mechanism that creates the ensemble. But maybe that mechanism is of a different kind than usually assumed - perhaps once it has found a successful model universe, it then churns out innumerable identical copies of the same universe.

In the end belief in a multiverse may always be just that – a matter of faith, namely faith that the logical arguments discussed here give the correct answer in a situation where direct observational proof is unattainable and the supposed underlying physics is untestable, unless we are able to point to compelling reasons based on scientifically supportable evidence for a particular specifiable multiverse or one of a narrowly defined class of multiverses. One way in which this could be accomplished, as we have already indicated, would be to find accumulating direct or indirect evidence that a very definite inflaton potential capable of generating a certain type of ensemble of universe domains was operating in the very early universe, leading to the particular physics that we observe now. Otherwise, there will be no way of ever knowing which particular multiverse is realised, if any one is. We will always be able to claim whatever we wish about such an ensemble, provided it includes at least one universe that admits life.

7.3 Observations and Physics

One way one might make a reasonable claim for existence of a multiverse would be if one could show its existence was a more or less inevitable consequence of well-established physical laws and processes. Indeed, this is essentially the claim that is made in the case of chaotic inflation. However the problem is that the proposed underlying physics has not been tested, and indeed may be untestable. There is no evidence that the postulated physics is true in this universe, much less in some pre-existing metaspace that might generate a multiverse.

Thus there are two further requirements which must still be met, once we have proposed a viable ensemble or multiverse theory. The first is to provide some credible link between these vast extrapolations from presently known physics to physics in which we have some confidence. The second is to provide some at least indirect evidence that the scalar potentials, or other overarching cosmic principles involved, really have been functioning in the very early universe, or before its emergence. We do not at present fulfil either requirement.

The issue is not just that the inflaton is not identified and its potential untested by any observational means - it is also that, for example, we are assuming quantum field theory remains valid far beyond the domain where it has been tested, and where we have faith in that extreme extrapolation despite all the unsolved problems at the foundation of quantum theory, the divergences of quantum field theory, and the failure of that theory to provide a satisfactory resolution of the cosmological constant problem.

7.4 Observations and disproof

Despite the gloomy prognosis given above, there are some specific cases where the existence of a chaotic inflation (multi-domain) type scenario can be disproved. These are when we either live in a universe with compact spatial sections because they have positive curvature, or in 'small universe' where we have already seen right round the universe (Ellis and Schreiber 1986, Lachieze-Ray and Luminet 1995), for then the universe closes up on itself in a single FLRW-like domain, and so no further such domains that are causally connected to us in a single connected spacetime can exist.

As regards the first case, the best combined astronomical data at present (from the WMAP satellite together with number counts and supernova observations) suggest that this is indeed the case: they indicate that $\Omega_0 = 1.02 \pm 0.02$ at a 2-σ level, on the face of it favoring closed spatial sections and a spatially finite universe. This data does not definitively rule out open models, but it certainly should be taken seriously in an era of 'precision cosmology.'

As regards the 'small universe' situation, this is in principle observationally testable, and indeed it has been suggested that the CBR power spectrum might already be giving us evidence that this is indeed so, because of its lack of power on the largest angular scales (Luminet et al, 2003). This proposal can be tested in the future by searching for identical circles in the CMB sky (Roukema, et al., 2004) and alignment of the CMB quadrupole and octopole planes (Katz and Weeks 2004). Success in this endeavour would disprove the usual chaotic inflationary scenario, but not a true multiverse proposal, for that cannot be shown to be false by any observation. Neither can it be shown to be true.

8 Special or Generic?

When we reflect on the recent history of cosmology, we become aware that philosophical predilections have oscillated from assuming that the present state of our universe is very special (made cosmologically precise in contemporary cosmology as FLRW, or almost-FLRW, through the assumption of a Cosmological Principle – see Bondi 1960 and Weinberg 1972, for example), requiring very finely tuned initial conditions, to assuming it is generic, in the sense that it has attained its present apparently special qualities through the operation of standard physical processes on any of broad range of possible initial conditions (e. g., the "chaotic cosmology" approach of Misner (1968) and the now standard but incomplete inflationary scenario pioneered by Guth (1980)). This oscillation, or tension, has been described and discussed in detail, both in its historical and in its contemporary manifestations, by McMullin (1993) as a conflict or tension between two general types of principle – anthropic-like principles, which recognize the special character of the universe and tentatively presume that its origin must be in finely tuned or specially chosen initial conditions, and "cosmogonic indifference principles," or just "indifference principles," which concentrate their search upon very generic initial conditions upon which the laws of physics act to produce the special cosmic configuration we now enjoy. As McMullin portrays these two philosophical commitments, the anthropic-type preference inevitably attempts to involve mind and teleology as essential to the shaping of what emerges, whereas the indifference-type preference studiously seeks to avoid any direct appeal to such influences, relying instead completely upon the dynamisms (laws of nature) inherent in and emerging from mass-energy itself.

McMullin (1993), in a compelling historical sketch, traces the preference for the special and the teleologically suggestive from some of the earlier strong anthropic principle formulations back through early Big Bang cosmology to Clarke, Bentley, William Derham and John Ray in the 18th and 19th centuries and Robert Boyle in the 17th century and ultimately back to Plato and the Biblical stories of creation. The competing preference for indifferent initial conditions and the operation of purely physical or biological laws can be similarly followed back from the present appeal to multiverses to slightly earlier inflationary scenarios and Misner's chaotic cosmology program to steady state cosmological models and then back through Darwin to Descartes and much earlier to the Greek atomists, such as Empedocles, Diogenes Laertius and Leucippus. Neither of these historical sequences involves clearly dependent philosophical influences, but the underlying basic assumptions and preferences of each of the two sets of thinkers and models are very similar, as are their controversies and interactions with the representatives of the competing approach.

Certainly it has become clear that the present preference among theoretical cosmologists for multiverse scenarios is the latest and most concerted attempt to implement the indifference principle in the face of the mounting evidence that, taken alone, our universe does require very finely tuned initial conditions. The introduction of inflation was similarly motivated, but has encountered some scepticism in this regard with the growing sense that initiating inflation itself probably requires special conditions (Penrose 1989; Ellis, et al. 2002). The appeal to multiverses, though first seriously suggested fairly early in this saga (by Dicke in 1961 and by Carter in 1968), has been reasserted as this failure of other indifference principle implementations seem more and more imminent. However, as we have just seen in our detailed discussion of realised ensembles of universes or universe domains, they are by no means unique, and accounting for their existence requires an adequate generating process or principle, which must explain the distribution function characterizing the ensemble.

Even though we are far from being able to connect specific types of ensembles with particular provisionally adequate cosmogonic generating processes in a compelling way, it is very possible that some fine-tuning of these processes may be required to mesh with the physical constraints we observe in our universe and at the same time to produce a realised ensemble which enbraces it. This would initiate another oscillation between the two types of principles. Whether or not that occurs, it is clear that the existence of a multiverse in itself does not support either the indifference principle nor the anthropic-type principle. What would do so would be the distribution function specifying the multiverse, and particularly the physical, pre-physical or metaphysical process which generates the multiverse with that distribution function, or range of distribution functions. Only an understanding of that process would ultimately determine which principle is really basic.

Whatever the eventual outcome of future investigations probing this problem, it is both curious and striking, as McMullin (1993, p. 385) comments, that "the same challenge arises over and over." Fine- tuning at one level is tentatively explained by some process at a more fundamental level which seems at first sight indifferent to any initial conditions. But then further investigation reveals that that process really requires special conditions, which demands some fine-tuning. Meanwhile, "the universe" required for understanding and explanation "keeps getting larger and larger."

It might seem that these competing philosophical or metaphysical preferences – for what is either basically special or basically generic – are choices without scientific or philosophical support. But that is an illusion. From what we have seen already, there is considerable physical and philosophical support for each preference – some of it observational and some of it theoretical – but there is no *adequate* or *definitive* support for one over against the other. Thus, either preference may be supported in various ways philosophically and scientifically, but neither the one nor the other is *THE* scientific approach. For example, the emergent universe model of Ellis, Maartens, et al. (2003) has fine-tuned initial conditions, but it still could be a good model – it may actually represent how the very early history of our universe unfolded, even though it does not explain how the special initial conditions were set. [In fact it is not as fine-tuned as inflation with $k = 0$, which requires "infinite" fine-tuning, while being "asymptotic" to an Einstein static universe does not.]

The issue is not so much which of the two principles or perspectives are correct – both seem to be important at different levels and in different heuristic and explanatory contexts. As far as we know, there has not been any resolution to the question of the epistemological or ontological status of either one. They function rather as contrary heuristic preferences which have both intuitive and experiential support. Perhaps the real question is: Which is more fundamental? It is possible that, from the point of view of physics, the indifference principle is more fundamental – relative to the explanations which are possible within the sciences – whereas from the point of view of metaphysics, an anthropic-type fine-tuning principle is more fundamental.

What does seem clear, in this regard, is that the effort to keep explanation and understanding completely within the realm of physics forces us to choose the indifference principle as more fundamental. This is simply because the need for fine-tuning threatens to take us outside of where physics or any of the other natural sciences can go. Furthermore, as we have also seen, physics and the other sciences cannot delve into the realm of ultimate explanation either.

9 Conclusion

As we stressed in the conclusion of EKS, the introduction of the multiverse or ensemble idea is a fundamental change in the nature of cosmology, because it aims to challenge one of the most basic aspects of standard cosmology, namely the uniqueness of the universe (see Ellis 1991, 1999 and references therein). So far, research and discussion on such ensembles have not precisely specified what is required to define them, although some specific physical calculations have been given based on restricted low-dimensional multiverses.

Our fundamental starting point has been the recognition that there is an important distinction to be made between possible universes and realised universes, and a central conclusion is that a really existing ensemble or multiverse is not *a priori* unique, nor uniquely defined. It must somehow be selected for. We have defined both the ensemble of possible universes $\mathcal{M}$, and ensembles of really existing universes, which are envisioned as generated by a given primordial process or action of an overarching cosmic principle, physical or metaphysical. This effectively selects a really existing multiverse from $\mathcal{M}$, and, as such, effectively defines a distribution function over $\mathcal{M}$. Thus, there is a definite causal connection, or “law of laws, ”relating all the universes in each of those multiverses. It is such a really existing ensemble of universes, one of which is our own universe, *not* the ensemble of all possible universes, which provides the basis for anthropic arguments. Anthropic universes lie in a small subset of $\mathcal{M}$, whose characteristics we understand to some extent. It is very likely that the simultaneous realisation of *all* the conditions for life will pick out only a very small sector of the parameter space of all possibilities: anthropic universes are fine-tuned in that sense. If cosmogonic processes or the operation of a certain primordial principle selected and generated an ensemble of really existing universes from $\mathcal{M}$, some of which are anthropic, then, though we would require some explanation for that process or principle, the fine-tuning of our universe would not require any other scientific explanation. It is, however, abundantly clear that “really existing ensembles” are *not* unique, and neither their properties nor their existence are directly testable. Arguments for their existence would be much stronger if the hypotheses employing them were fruitful in enabling new investigations leading to new predictions and understandings which are testable. However, so far this has not been the case. In our view these questions - Issues 1 and 2 in this paper – cannot be answered scientifically with any adequacy because of the lack of any possibility of verification of any proposed underlying theory. They will of necessity have to be argued with a mixture of careful philosophically informed science and scientifically informed philosophy. And, even with this, as we have just seen, we seem to fall short of providing satisfactory answers – so far!

Another philosophical issue we have emphasized which has a strong bearing on how we describe and delimit really existing multiverses is that of realised infinity. From our careful discussion of this concept, there is a compelling case for demanding that every really existing ensemble contain only a finite number of universes or universe domains.

There is strong support for both of two competing approaches – that which honors the special character of our universe by stressing the need for the fine-tuning of initial conditions and the laws of nature, and that which locates its emergence in the operation of primordial processes on a much more fundamental generic or indifferent configuration. Both are undoubtedly at work on different levels. The issues are: which is more fundamental, and whether the sciences themselves as they are presently conceived and practiced can deal with ultimate fundamentals. Must they yield that realm to metaphysics? Can metaphysics deal with them? The relative untestability or unprovability of the multiverse idea in the usual scientific sense is however problematic – the existence of the hypothesized ensemble remains a matter of faith rather than of proof, unless it comes to enjoy long-term fruitfulness and success. Furthermore in the end, the multiverse hypothesis simply represents a regress of causation. Ultimate questions remain: Why this multiverse with these properties rather than others? What endows these with existence and with this particular type of overall order? What are the ultimate boundaries of possibility – what makes something possible, even though it may never be realised?

As we now see, the concept of a multiverse raises many fascinating issues that have not yet been adequately explored. The discussions here should point and guide research in directions which will yield further insight and understanding.

Acknowledgements

We thank A. Aguirre, C. Harper, A. Lewis, A. Linde, A. Malcolm, and J.P. Uzan for helpful comments and references related to this work, and two anonymous referees for comments and suggestions which have enabled us to correct errors and clarify our positions. GFRE and UK acknowledge financial support from the University of Cape Town and the NRF (South Africa).

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